

## 1.3. Co-channel Interference and System Capacity

### Reading Assignment

- Chapters 3.5, 3.7.

## 1 Co-channel Interference

In section 1.2, system capacity, defined as the total channels supported in a cellular system is derived using simple cellular geometry. It was shown that through frequency reuse system capacity can be improved dramatically (seemingly unbounded!). The analysis is somewhat naive — it does not take into account the serviceability of a channel.

Intimately coupled with frequency reuse is the phenomenon called co-channel interference. This is because the same frequency band is used by two or more base stations that are located in relative proximity to each other. Even though cellular structure is designed to provide maximum protection against interference, it simply can not totally eliminate co-channel interference, which in some cases can be the dominant factor in determining the system capacity (See HW#1 Problem 4). Having a clear understanding of the impact of co-channel interference on the system capacity is therefore of great importance in system design.

Generally speaking, a channel is deemed serviceable if its signal to interference/noise ratio (SINR) is above a certain threshold<sup>†</sup>. A system is called interference limited if the co-channel interference dominates the channel noise. Under this assumption, we can assume a noiseless channel to simplify our analysis and gain valuable insight on the effects of interference. Notice that co-channel interference exists for both downlink and uplink channels and we will use downlink in the sequel and relegate uplink analysis in your homework (Hw#1, Problem 2).

## 2 Approximate Analysis

Under noiseless assumption, the signal to interference ratio can be written as

$$\frac{S}{I} = \frac{S}{\sum_{i=1}^{i_0} I_i}$$

where

$S$  received signal power from the desired base station;

$I_i$  interference power caused by the  $i^{\text{th}}$  interfering co-channel cell base station;

$i_0$  total number of co-channel cells (interfering BS).

A thorough and accurate analysis is almost impossible except for some special case (See reference [Jac94] as well as page 70 in the textbook). However, when  $N$  is relatively large ( $\geq 7$ ), a simple first-tier approximation may be sufficient. Recall the procedure for locating co-channel cells introduced

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<sup>†</sup>In reality, the actual signal strength needs to be also considered. This is especially true when noise, in addition to co-channel interference, is considered.

in the last section. We call those co-channel cells that are immediate neighbors of the reference cells as first-tier co-channel cells. Similarly, repeating the procedure starting from first-tier cells, we can reach second-tier co-channel cells, etc. If  $N$  is large, then first tier co-channel interference will dominate therefore may suffice in analysis.

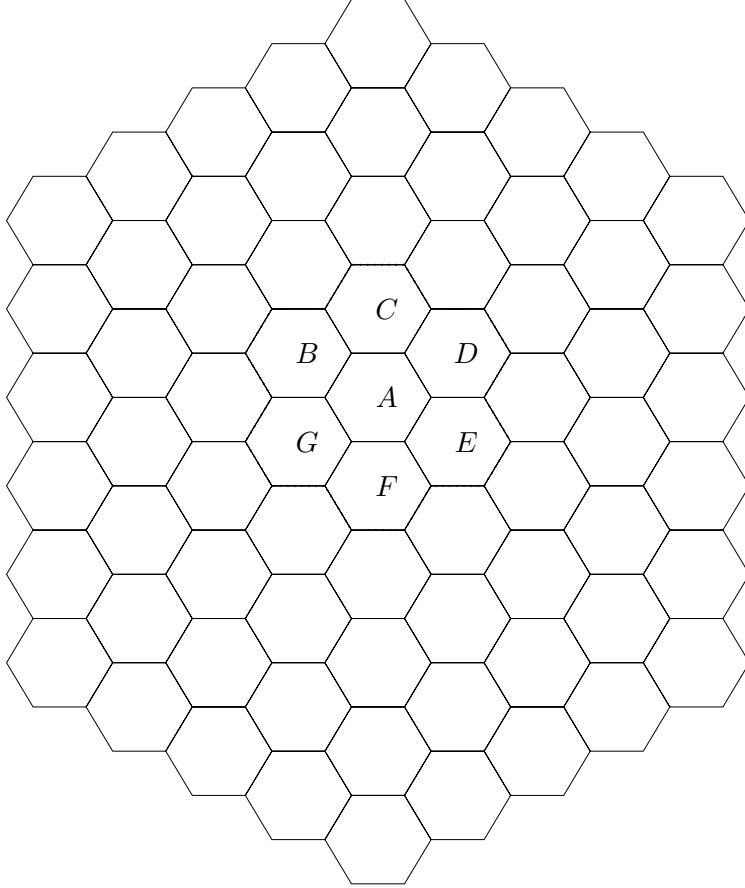


Figure 1: Hexagon cellular structure with  $N = 7$ .

Try to locate co-channel cells for cell  $A$  in Figure 1. The worst case scenario is when a MS is located at one of the vertices of the cell hence its distance to the desired BS is the cell radius  $R$ . Using crude approximation (valid when  $N$ , hence  $D/R$  is large), we assume for now that the distance between the MS and the first tier co-channel BS are all equal to  $D$ . Given the power-law decaying of the signal power with a path loss exponent  $n$ , the SIR can be approximated using

$$\frac{S}{I} = \frac{R^{-n}}{\sum_{i=1}^{i_0} D_i^{-n}}$$

Here  $i_0$  is the number of first-tier interfering BS and is 6 for the omnidirectional antenna (more on this when we talk about sectoring). Using the result that  $Q = D/R = \sqrt{3N}$ , we have

$$\frac{S}{I} = \frac{1}{6}(\sqrt{3N})^n$$

Clearly the cluster size  $N$  is indeed related to system capacity as it directly affects the SIR.

**Example 1** Assume that the SNR threshold for serviceable channel is 15dB and that the path loss exponent  $n = 4$ . Determine the minimum cluster size  $N$  that satisfies the hexagonal cellular geometry. What if we raise the SIR threshold to 20dB?

The reason that larger  $N$  results in an improved SIR performance is because by increasing  $N$ , we also increase the relative distance from the MS to the interfering BS (there are more cells in a cluster!). This implies the nice scalability property: SIR does not depend on the area of a cell (though the actual carrier strength and interference strength do!).

Improving SIR through increasing  $N$ , however, has limited value in practice. First, large  $N$  means smaller cells and therefore more frequent handoffs which may be infeasible. Furthermore the cost of BS is often the most critical factor in system design. Often times because of above concerns, we prefer a fixed cell size. In this case, it turns out that the smaller the  $N$  can be, the higher the system capacity is. To understand this, consider the extreme case of  $N = 1$  (though it is impractical because of excessive interference).

Below is a system design exercise that helps you to understand the tradeoff among different parameters.

**Example 2** Assume we need to design a cellular system that covers an area of radius  $R = 20$  miles. The total number of channels allocated to the system is 200 with a target system capacity of 2000 (i.e., we need to provide service to 2000 users simultaneously). The minimum C/I threshold is 20dB with a path loss exponent  $n = 4$ . Determine the maximum cell size.

Users are evenly distributed geographically. Further, hexagonal cellular geometry with first tier C/I approximation is used.

**Exercise 1** Assume we need to design a cellular system that covers an area of radius  $R = 20$  miles. For practical reasons (cost of BS, handoff capability, etc.), each cell should cover at least an area of radius  $r = 1$  mile. FDD is used and that each simplex channel requires a bandwidth of 25kHz. The minimum C/I threshold is 20dB with a path loss exponent  $n = 4$ . Assuming that the target system capacity is 2000 (i.e., we need to provide service to 2000 users simultaneously), find the minimum total bandwidth needed.

Users are evenly distributed geographically. Further, hexagonal cellular geometry is assumed and first-tier C/I approximation is deemed sufficient.

One of the lucky folks will need to present his/her design strategy next class.

### 3 Improved Approximation for $N = 7$

In reference to Figure 3.5 in the textbook, the interfering first-tier BS can be divided into three groups, each with approximate distance  $D - R$ ,  $D + R$ , and  $D$ . Hence

$$\frac{S}{I} = \frac{R^{-n}}{2(D - R)^{-n} + 2(D + R)^{-n} + 2D^{-n}}$$

$$= \frac{1}{2(Q-1)^{-n} + 2(Q+1)^{-n} + 2Q^{-n}}$$

Using this approximation for  $n = 4$ , it turns out that  $N > 5.61$  for 15dB SIR threshold while  $N > 9.22$  for 20dB. Previous first-tier approximation is too optimistic.

## 4 Capacity improvement

### 4.1 Cell splitting

For a fixed coverage area, if we keep  $N$  constant (hence keeping interference at check) but decrease the size of each cell, then the total number of channels supported in the system will increase as the number of cells will increase. This is the basic principle behind cell-splitting.

The term cell-splitting is usually referring to the techniques used in situations where existing system has reached its capacity due to the increasing number of subscribers along the years. In that case, for those cells that are saturated with user traffics are then split into smaller cells to provide more channels. See Figure 3.8 in the text and the discussion therein.

We mention here that another technique that can alleviate the saturation of certain cells (uneven traffic within the whole area) in the system is the dynamic channel allocation technique.

### 4.2 Sectoring

While cell splitting improves the capacity by in essence adding more BS, a different approach called sectoring can achieve similar goal through the control of co-channel interference. Instead of using an omnidirectional antenna at a BS, we can use several directional antenna (e.g., three 120° sectoring antenna or six 60° sectoring antenna) to cover the same cell. The total channels assigned to the cell are split among the sectors. In this way, a BS will see less number of interfering BS because of the directiveness of the antenna. See Figure 3.11 in the text and the discussion therein

**Example 3** Assume  $N = 7$  and  $n = 4$ . Calculate the SIR using first-tier approximation assuming (1) 120° sectoring, and (2) 60° sectoring. Compare that with the result for no sectoring.

The fact that sectoring alleviate the co-channel interference allows the use of smaller  $N$  therefore improve the frequency reuse efficiency and system capacity. The negative effect is the decreased trunking efficiency as we shall see later.