1.4. GOS and Trunking Efficiency

Reading Assignment

- 3.6.
- Optional: Appendix A.1.

1 Trunking

In section 1.3, we consider system capacity from the perspective of simultaneously available channels in the system. This does not translate into the actual number of users supported in the system — it is rarely the case that all users within a system will access the system (i.e., using their cell phone) at the same time. The fact that users access the system in a random and independent fashion is used carefully by the engineers so that a small number of channels can support a potentially large number of users — the concept of trunking. Simply put, trunking system utilizes the statistical behavior of users to dynamically assign channels to users on demand so that the total number of users accommodated in a system can be substantially larger than the number of channels. We note in passing that trunking is not a new concept for mobile communications — it has been used for years in the design of telecommunication systems to provide efficient and cost effective service to users.

To quantitatively determine the maximum number of users allowed in a system, we need first to specify a certain measure for grade of service. Notice that by allowing the number of users, denoted by U, to exceed the number of channels provided, denoted by C, we are exposed to the risk that at some instance some users may be denied service because all C channels are occupied. Since from the user's end, it is desirable to have the chance of service denial at a minimum, the probability that a user is denied access to the channels is used as a measure for grade of service (GOS). This probability is usually called blocking probability in telephone networks.

There are two types of trunking. The first is called the *blocked call cleared* (BCC, also called lost call cleared, or LCC). The second is *blocked call delayed* (BCD). In BCC trunking system, if a user is denied service, he/she will terminate his/her service request and go back to inactive status. In BCD system, a queue is provided to buffer denied calls and the service will be available after some delay (when some busy channels are freed up). We will only consider BCC. For BCD system, please refer to the text and the reference therein.

2 Grade of Service

If the channel number, C, is smaller than the total number of users in the system, U, there is a chance that at some instance (though should be deemed rare for a practical system) the system is operating at full capacity, i.e., all channels are used. At this point, any users who attempt to access the channel will be denied service.

The probability of service denial, or blocking probility, is a function of channel number, offered traffic, as well as the channel grouping scheme. A quantitative analysis is presented next.

2.1 Traffic Intensity

Traffic intensity measures the intensity of user activity. From a different point of view, it measures how often the system channels are occupied. Its unit, Erlang (Due to A. K. Erlang of Denmark), represents the amount of traffic intensity carried by a channel that is completely occupied. For example, if a user is on the phone 20% of the time, then the traffic offered by that user is 0.2 Erlang. From the perspective of channel utilization, if a channel is occupied thirty minutes out of an hour, then the traffic it carries is 0.5 Erlang.

Clearly we see that the offered traffic and carried traffic are two different concepts, especially in a trunking system. Offered traffic refers to the traffic that the users generate in the system while carried traffic is the total traffic serviced by the channels. If a system has unlimited capacity, then the offered traffic always equals to carried traffic. In system with limited capacity, if the offered traffic exceeds the system capacity, the system would only be able to carry the traffic equal to that of its capacity.

Assume that for a particular user, the average duration of a call is H and the average number of call requests per unit time is λ, then the total traffic intensity offered by this user is

$$A_u = \lambda H$$

Further assume that the users in a system has the same traffic intensity, then a system with U users has a total offered traffic intensity

$$A = UA_u$$

For a trunked system with C channels (hence the maximum carried traffic is C erlangs), the average traffic intensity per channel is

$$A_c = UA_u/C$$

Larger A_c results in higher blocking probability.

Example 1 *What is the offered traffic for a system with* 300 *users, each user generating a call at the rate of one call every two hours with average call duration 3 minutes?*

2.2 Erlang loss formular

To quantitatively determine the blocking probability, some statistical assumptions must be made regarding the patterns of user call request as well as the duration of each call. Erlang loss formular, also called Erlang B formular, is based on the M/M/m/m queque model. Its derivation can be found in standard queueing theory books as well as in Appendix A. The assumptions used in deriving the formular are:

1. Infinite user. This is reasonable for large trunking system.
2. Memoryless call arrival: Poisson.
3. Exponential service time.
4. Finite number of channels with BCC (blocked call cleared).

Under these assumption, it is derived that the blocking probability is

$$Pr[Blocking] = \frac{A^C/C!}{\sum_{k=0}^{C} A^k/k!}$$

where A is the total offered traffic and C is the number of channels in the trunking system. An easy way to remember and understand the formular is to notice that a Poisson process has probability of number of arrivals per unit time

$$Pr[K = k] = \frac{A^k}{k!}e^{-A}$$

Thus we can rewrite the blocking probability as

$$Pr[Blocking] = \frac{e^{-A}A^C/C!}{\sum_{k=0}^{C} e^{-A}A^k/k!} = \frac{\text{Probability of having } C \text{ arrivals}}{\text{Probability of having up to } C \text{ arrivals}}$$

which is the probability of running at full capacity (all C channels are used).

Example 2 *Assume that the offered traffic is as in Example 1. For a GOS of 2%, how many channels are needed? What if we raise the GOS to 1%?*

Keep in mind that the offered traffic here is the statistical average of the actual traffic at any moment, with the actual traffic volume being a random variable (or a random process, to be exact). Thus the actual traffic volume will vary from time to time. The probability that this random variable will exceed the system capacity is the blocking probability. See Figure 1 for clarification.

2.3 Trunking efficiency

A measure of efficiency for a trunked system is the maximum traffic intensity that is carried by the system subject to a given GoS. Since the total traffic is obviously a function of the number of channels, we usually need to define clearly the context within which we define the trunking efficiency. For example, for cellular system with a fixed number total assigned channels, different N will result in different channels per cell therefore different trunking efficiency per cell.

Table 3.4 in the text book is a list of trunking efficiency for certain channel numbers, offered traffic, and GOS specifications. We can immediately see the impact of channel grouping (think cellular system) on the trunking efficiency. The capacity for $C = 10$ with GOS 0.01 is 4.46 Erlangs, while the apacity for $C = 70$ with GOS 0.01 is 56.1 Erlangs. Therefore if 70 channels is divided into 7 groups with 10 channel each, there is a loss of capacity at

$$\frac{56.1 - 7 \times 4.46}{56.1} = 44.35\%$$

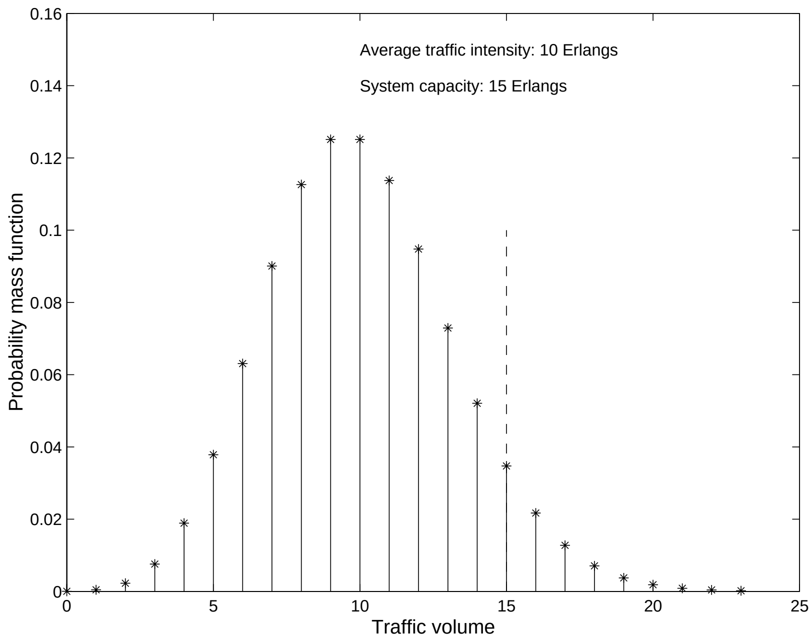

Figure 1: Plotted is the probability mass function of traffic volume given an average traffic intensity of 10 Erlangs. The blocking probability is the sum of probability mass that exceeds (to the right of) the system capacity which is 15 Erlangs in the plot.

This does not mean, however, that the cellular system decreases the trunking efficiency. First, the total number of channels increases due to the frequency reuse. Second, for a given cellular system, different channel assignment scheme will result in different trunking efficiency. For example, dynamic channel assignment, i.e., channels are assigned dynamically in accordance with user traffic, has better trunking efficiency than fixed channel assignment scheme.

On the other hand, sectoring, aimed to decrease the co-channel interference, will indeed decrease trunking efficiency.

Example 3 *A cellular system with* $N = 12$ *has* 100 *channels per cell. If the desired* $Pr[Blocking] = 2\%$ *with* $A_u = 0.1$*, find the trunking efficiency. If* 120^o *sectoring is used, what will be the trunking efficiency? How many users can each cell support in both cases?*