Decision Fusion Rules in Multi-Hop Wireless Sensor Networks

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The decision fusion problem for a wireless sensor network (WSN) operating in a fading environment is considered. In particular, we develop channel-aware decision fusion rules for resource-constrained WSNs where binary decisions from local sensors may need to be relayed through multi-hop transmission in order to reach a fusion center. Each relay node employs a binary relay scheme whereby the relay output is inferred from the channel impaired observation received from its source node. This estimated binary decision is subsequently transmitted to the next node until it reaches the fusion center. Under a flat fading channel model, we derive the optimum fusion rules at the fusion center for two cases. In the first case, we assume that the fusion center has knowledge of the fading channel gains at all hops. In the second case, we assume a Rayleigh fading model, and derive fusion rules utilizing only the fading channel statistics. We show that likelihood ratio (LR) based optimum decision fusion statistics for both cases reduce to respective simple linear test statistics in the low channel signal-to-noise ratio (SNR) regime. These suboptimum detectors are easy to implement and require little a priori information. Performance evaluation, including a study of the robustness of the fusion statistics with respect to unknown system parameters, is conducted through simulations.

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I. INTRODUCTION

Wireless sensor networks (WSN) have generated enormous interest from researchers in various disciplines. Current and future applications range from battlefield surveillance, health care and telemedicine, to environmental and habitat monitoring and control [1–6].

A distinct feature of WSNs is that wireless communication networks become an integral component of the WSN. This is especially true for resource-constrained WSN where a divide and conquer approach (i.e., treating the communication network as an independent entity) may lead to significantly inferior performance as well as potential wastage of limited resources. The integration of information transmission and processing appears to be a promising direction for optimized system performance under given resource constraints. Of particular concern in the work reported here is the information fusion task at the fusion center. For a distributed detection system, the conventional way is to consider communication and decision fusion as two independent parts, and to design them separately. Numerous results on the classical distributed detection problems were obtained during the past decades. In [7] and [8], optimum fusion rules have been investigated under the conditional independence assumption. Many papers have also addressed the problem of distributed detection with constrained system resources [9–13], most of which have provided the solutions to optimize bit allocation (or sensor selection) given a constraint on the total amount of communications. The above results, however, are mostly obtained based on the assumption of lossless communication (i.e., the information sent from the local sensors is perfectly recovered at the fusion center). This assumption is not realistic for many WSNs where the transmitted information has to endure both channel fading and noise/interference. This motivates the study of the fusion of local decisions corrupted by channel fading/noise impairment.

Decision fusion with nonideal communication channels has been studied at both the fusion center level [14–17] and at the sensor level [14, 18, 19]. In [14], Thomopoulos and Zhang derived the optimal thresholds both at the fusion center and the local sensors by assuming a simple binary symmetric channel between sensors and the fusion center. Their method is quite complex, and requires global knowledge of the entire system. Besides, the binary symmetric channel assumption does not allow a full integration of transmission into the decision fusion stage. In [15]–[17], channel-aware decision fusion rules have been developed using a canonical distributed detection system where binary decisions from multiple parallel sensors are transmitted through fading channels to a fusion center where they are combined for final decision making. The canonical fusion structure, while theoretically important and analytically tractable, may not reflect the way a real WSN operates. In most WSN applications, resource constraints, especially the energy constraint in applications involving in-situ unattended sensors operating on irreplaceable power supplies, often limit the transmission range for each sensor node. Radio transmission is one of the major power consumers among all the functions for a sensor node, and the required transmission power is not linear in distance between the transmitter and the receiver. Hence, in order to reach a fusion node, a decision made at a local sensing node may need to go through multiple hops for minimal energy consumption.

The objective of this work is to extend the channel-aware decision fusion rules developed in [15]–[17] to more realistic WSN models that involve multi-hop transmissions. We present a theoretical formulation of the multi-hop decision fusion problem and design new fusion rules for the case where binary local decisions are relayed to a fusion center through multi-hop wireless channels. For multi-hop transmissions, the relay nodes are to convey the information received from their source nodes to their destination nodes. Under an ideal situation, each relay node recovers the original decision correctly; hence the fact that local decisions undergo multiple hops does not have any impact on the fusion center design. However, the assumption of reliable relaying is overly optimistic in light of the limited resources and stringent delay requirement, as well as the potential severity of channel impairment. Thus, in this work, we assume a simple memoryless relay scheme where each relay node decides what to transmit using its own observation by employing a maximum likelihood (ML) estimate. The estimated decision may be inconsistent with what was originally transmitted and this has to be taken into account in the fusion rule design. Given the above binary relay scheme and assuming a flat fading channel model, we derive the optimal decision fusion algorithms for the following two cases. In the first case, we assume that the fading channel gains for all the hops are available at the fusion center and derive the optimal likelihood ratio (LR) based fusion statistic. In the second case, we relax the requirement to knowing only the fading channel statistics by assuming a Rayleigh fading channel model and derive the corresponding optimal LR fusion rule. We emphasize that the flat fading channel assumption is valid for many WSN applications where sensors employ a low rate transmission scheme (hence large symbol interval) and the fact that they are densely deployed in an open field, resulting in a small delay spread.

The LR-based optimal fusion rules obtained for both cases still require a significant amount of prior information that is either not available or, in some cases, can only be acquired at a cost level that is not permissible for real WSN systems. For example, for fast-fading channels, channel envelopes may only remain constant for a single channel use. Thus, each relay node may need to send the channel envelope information to either its next level relay or the fusion center during each relay transmission. This will increase the system overhead that cost extra resources. Therefore, fusion rules that do not rely on channel coefficients would be practical. As such, by imposing additional assumptions, we further reduce the LR-based fusion rules to some simple linear test statistics in the low signal-to-noise ratio (SNR) regime. As it turns out, both low SNR alternatives deemphasize the sensors with more hops and are in the form of a weighted sum of the channel outputs, where the weight is a function of the product of all link SNRs along each relay path. We also show that the Chair-Varshney fusion rule [7] provides a high SNR approximation to the LR-based fusion rules in both cases.

The organization of the paper is as follows. In the next section, we review the previous work on fusion rule design for a canonical parallel distributed detection system with single hop transmission between sensor nodes and the fusion center. In Section III, we lay out the model for a multi-hop based sensor fusion network. The case of known fading channel amplitude is treated in Section IV, followed by the case where only the fading channel statistics are known in Section V. Performance evaluation is given in Section VI and we conclude in Section VII.

II. REVIEW OF PREVIOUS WORK

Fig. 1 depicts a typical parallel fusion structure where a flat fading channel model is assumed between each sensor and the fusion center. K sensors collect data generated according to either H_0 or H_1, the two hypotheses under test, make local decisions, and transmit these decisions over fading and noisy channels to a fusion center. The fusion center tries to decide which hypothesis is true based on the received data y_k for all k. Assume that the kth local sensor makes a binary decision $u_k \in \{+1,-1\}$, with false alarm and detection probabilities P_{fk} and P_{dk}, respectively. That is, we have $P[u_k = 1 \mid H_0] = P_{fk}$ and $P[u_k = 1 \mid H_1] = P_{dk}$. The received signal at the fusion center from the kth sensor is

$$y_k = h_k u_k + n_k \qquad (1)$$

where h_k is the channel fading envelope and n_k is zero-mean additive Gaussian noise with variance σ^2. Using the above fusion model, we can obtain the following set of five decision fusion rules, depending on the amount of prior knowledge available [15, 16].

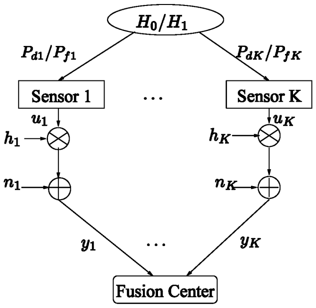


Fig. 1. Canonical single-hop fusion model in presence of fading channels.

Throughout this work, we use $\Lambda^{(s)}$ to denote the fusion statistics for the single hop transmission model in order to distinguish from that for the multi-hop systems.

1) Optimal LR-based fusion statistic using complete prior knowledge. Assuming complete channel knowledge, the optimal LR-based fusion statistic was derived in [15] and [17]

$$\Lambda_1^{(s)} = \prod_{k=1}^{K} \frac{P_{dk} e^{-((y_k - h_k)^2/2\sigma^2)} + (1-P_{dk}) e^{-((y_k + h_k)^2/2\sigma^2)}}{P_{fk} e^{-((y_k - h_k)^2/2\sigma^2)} + (1-P_{fk}) e^{-((y_k + h_k)^2/2\sigma^2)}} \tag{2}$$

where $\mathbf{y} = [y_1, \ldots, y_K]^T$ is a vector containing observations received from all K sensors.

2) LR-based fusion rules using only fading statistics for Rayleigh fading channel. Implementing the optimal LR test as in (2) requires that all a priori information, including the instantaneous channel gains, is available. Under the Rayleigh fading model, the LR-based fusion statistic using only the fading parameter is summarized below [16].

THEOREM 1 *The LR for decision fusion under the Rayleigh fading channel model is*

$$\Lambda_2^{(s)} = \prod_{k=1}^{K} \frac{P_{dk}\left[1 + \sqrt{2\pi} t y_k e^{(t^2 y_k^2/2)} Q(-y_k t)\right] + (1-P_{dk})\left[1 - \sqrt{2\pi} t y_k e^{(t^2 y_k^2/2)} Q(t y_k)\right]}{P_{fk}\left[1 + \sqrt{2\pi} t y_k e^{(t^2 y_k^2/2)} Q(-y_k t)\right] + (1-P_{fk})\left[1 - \sqrt{2\pi} t y_k e^{(t^2 y_k^2/2)} Q(t y_k)\right]} \tag{3}$$

where $t = (\sigma_c/\sigma_n \sqrt{\sigma_c^2 + \sigma_n^2})$ *with* $2\sigma_c^2$ *being the mean square value of the fading channel,* σ_n^2 *is the noise variance, and* $Q(\cdot)$ *is the complementary distribution function of a standard Gaussian random variable.*

3) A two-stage approximation using the Chair-Varshney fusion rule. A direct alternative to the above LR-based fusion rules is to consider the information transmission and decision fusion as a two-stage process: first y_k is used to infer about u_k; then, the estimates of u_k are employed in the optimum fusion rule. Given the model in (1), the ML estimate for u_k is $\hat{u}_k = \text{sign}(y_k)$. Applying the fusion rule derived in [7], herein termed the Chair-Varshney fusion rule, we obtain the following statistic

$$\Lambda_3^{(s)} = \sum_{y_k<0} \log\left(\frac{1-P_{dk}}{1-P_{fk}}\right) + \sum_{y_k>0} \log\left(\frac{P_{dk}}{P_{fk}}\right). \tag{4}$$

Not surprisingly, $\Lambda_3^{(s)}$ can be shown to be mathematically equivalent to the two LR-based fusion rules in the large SNR regime (i.e., $\sigma^2 \to 0$) [15, 16].

4) Fusion statistic using a maximum ratio combiner (MRC). In the low SNR regime($\sigma^2 \to \infty$), we can show that $\Lambda_1^{(s)}$ reduces to $\hat{\Lambda}_4^{(s)} = \sum_{k=1}^{K} (P_{dk} - P_{fk}) \cdot h_k y_k$. Further, if the local sensors are identical, i.e., P_{dk} and P_{fk} are the same for all ks, then $\Lambda_1^{(s)}$ reduces to a form analogous to an MRC

$$\Lambda_4^{(s)} = \frac{1}{K} \sum_{k=1}^{K} h_k y_k. \tag{5}$$

5) Fusion statistic using an equal gain combiner (EGC). At low SNR($\sigma^2 \to \infty$), $\Lambda_2^{(s)}$ in (3) reduces to $\hat{\Lambda}_5^{(s)} = \sum_{k=1}^{K} (P_{dk} - P_{fk}) \sqrt{2\pi} t y_k$. Further, if the local sensors are identical, i.e., P_{dk} and P_{fk} are the same for all ks, then $\Lambda_2^{(s)}$ further reduces to a form analogous to an EGC [16]

$$\Lambda_5^{(s)} = \frac{1}{K} \sum_{k=1}^{K} y_k. \tag{6}$$

Among the above five fusion rules, $\Lambda_1^{(s)}$, requires complete channel knowledge and provides uniformly the most powerful detection performance. At low SNR, the MRC statistic provides the best performance among the three suboptimum fusion rules; while at high SNR, the Chair-Varshney fusion rule outperforms the MRC and the EGC statistics. The EGC statistic, however, provides better performance over a wide range of SNR than the MRC statistic and the Chair-Varshney fusion rule and requires the least amount of prior information.

III. MULTI-HOP DECISION FUSION PROBLEM

Consider a decision fusion network with multi-hop transmissions as illustrated in Fig. 2. Each sensing node observes data generated according to one of the two hypotheses under test, makes a local decision, and transmits the decision to a fusion center through several relay nodes. Each relay node tries to retrieve the decision sent from its source node from fading and noise impaired observation and relays it to the next node until it reaches the fusion center. The following assumptions are used in our analysis.

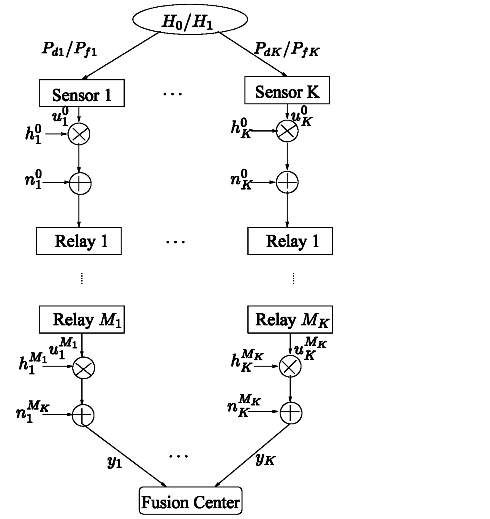


Fig. 2. Multi-hop parallel fusion model in presence of fading and noisy channels between local sensors and fusion center.

1) Binary transmission: all sensors (including local sensors and their relay nodes) make a binary decision which is either +1 or −1.

2) All the channels are independent of each other; and each of them can be modeled as a Rayleigh flat fading channel with identical mean squared value $2\sigma_c^2$, i.e., $E[(h_k^i)^2] = 2\sigma_c^2$, for $k = 1,2,\ldots,K$; $i = 0,1,\ldots,M_k$. Generalization to nonidentical channel statistics is fairly straightforward.

3) Noise processes on all the channels are Gaussian with zero mean and variance σ^2, and are independent of each other.

4) Phase coherent reception, hence the effect of a fading channel is simplified as a real scalar multiplication given that the transmitted signal is assumed to be binary.

5) Relay nodes do not directly observe the target.

With the above assumptions, we can formulate our multi-hop decision fusion network model as follows.

Suppose there are M_k relay nodes between the kth local sensor and the fusion node, then the number of hops from the kth local sensor to the fusion center is $M_k + 1$. Let u_k^0 denote the original binary decision of the kth local sensor, while u_k^i, $i = 1,2,\ldots,M_k$, denote the retrieved decisions corresponding to the ith relay node, where i is the hop index. Let P_{fk} and P_{dk} denote the false alarm and detection probability for the kth local sensor, i.e., $P[u_k^0 = 1 \mid H_0] = P_{fk}$ and $P[u_k^0 = 1 \mid H_1] = P_{dk}$, with $k = 1,2,\ldots,K$.

For each relay node, we assume a simple binary output which is the ML estimate of the decision sent from its source node. Hence, given that the noise is Gaussian, we have

$$u_k^i = \mathrm{sign}(u_k^{i-1} h_k^{i-1} + n_k^{i-1}).$$

Let y_k denote the input of the fusion center corresponding to the kth local sensor, thus,

$$y_k = u_k^{M_k} h_k^{M_k} + n_k^{M_k}$$

where h_k^i(a nonnegative real number) is the corresponding channel envelope and n_k^i is additive Gaussian noise with zero mean and variance σ^2.

Our goal is to derive fusion rules using y_k, $k = 1,2,\ldots,K$, that provide robust performance and, at the same time, require as little prior information as possible.

IV. FUSION RULES WITH KNOWN CHANNEL ENVELOPES

In this section, we start with the derivation of the LR-based fusion rules assuming known channel envelopes. Here "known channel envelopes" refers to the fact that the instantaneous channel envelope h_k^i, $k = 1,2,\ldots,K$; $i = 0,1,\ldots,M_k$ are all available at the fusion center. In Section V, we consider the case where only channel fading statistics are available.

A. The Optimum LR-Based Fusion Rule

Using the multi-hop fusion model as described in Fig. 2 and Section III, we now derive the LR-based fusion rule at the fusion center. First we introduce the notion of composite local performance indices, $P_{dk}^{(c)}$ and $P_{fk}^{(c)}$, defined as

$$P_{dk}^{(c)} = P(u_k^{M_k} = 1 \mid H_1)$$
$$P_{fk}^{(c)} = P(u_k^{M_k} = 1 \mid H_0).$$

They are the probabilities of declaring H_1 at the last relay when the true hypothesis is H_1 and H_0, respectively. This is different from the local performance indices P_{dk} and P_{fk}.

Given $P_{dk}^{(c)}$ and $P_{fk}^{(c)}$, the LR can be written as

$$\begin{aligned}\Lambda_1 &= \frac{f(\mathbf{y} \mid H_1)}{f(\mathbf{y} \mid H_0)} \\ &= \frac{\prod_{k=1}^K f(y_k \mid H_1)}{\prod_{k=1}^K f(y_k \mid H_0)} \\ &= \prod_{k=1}^K \frac{\sum_{u_k^{M_k}} f(y_k \mid u_k^{M_k}, H_1) P[u_k^{M_k} \mid H_1]}{\sum_{u_k^{M_k}} f(y_k \mid u_k^{M_k}, H_0) P[u_k^{M_k} \mid H_0]} \\ &= \prod_{k=1}^K \frac{P_{dk}^{(c)} e^{-((y_k - h_k^{M_k})^2/2\sigma^2)} + (1 - P_{dk}^{(c)}) e^{-((y_k + h_k^{M_k})^2/2\sigma^2)}}{P_{fk}^{(c)} e^{-((y_k - h_k^{M_k})^2/2\sigma^2)} + (1 - P_{fk}^{(c)}) e^{-((y_k + h_k^{M_k})^2/2\sigma^2)}} \\ &= \prod_{k=1}^K \frac{P_{dk}^{(c)} + (1 - P_{dk}^{(c)}) e^{-(2y_k h_k^{M_k}/\sigma^2)}}{P_{fk}^{(c)} + (1 - P_{fk}^{(c)}) e^{-(2y_k h_k^{M_k}/\sigma^2)}} \end{aligned} \quad (7)$$

where the assumption of conditional independence of local decisions is used.

Using the model specified in Section III, the two parameters $P_{dk}^{(c)}$ and $P_{fk}^{(c)}$ can be derived as follows. First, define

$$P_{1k}^{M_k} = P(u_k^{M_k} = 1 \mid u_k^0 = 1, H_1) = P(u_k^{M_k} = 1 \mid u_k^0 = 1)$$
$$P_{2k}^{M_k} = P(u_k^{M_k} = 1 \mid u_k^0 = -1, H_1) = P(u_k^{M_k} = 1 \mid u_k^0 = -1).$$

Hence,

$$\begin{aligned} P_{dk}^{(c)} &= P(u_k^{M_k} = 1 \mid H_1) \\ &= \sum_{u_k^0} P(u_k^{M_k} = 1 \mid u_k^0, H_1) P(u_k^0 \mid H_1) \\ &= P_{dk} P_{1k}^{M_k} + (1 - P_{dk}) P_{2k}^{M_k} \qquad (8) \\ P_{fk}^{(c)} &= P(u_k^{M_k} = 1 \mid H_0) \\ &= \sum_{u_k^0} P(u_k^{M_k} = 1 \mid u_k^0, H_0) P(u_k^0 \mid H_0) \\ &= P_{fk} P_{1k}^{M_k} + (1 - P_{fk}) P_{2k}^{M_k} \qquad (9) \end{aligned}$$

where $P_{1k}^{M_k}$ and $P_{2k}^{M_k}$ can be recursively determined as in the first part of Appendix A. Given $P_{dk}^{(c)}$ and $P_{fk}^{(c)}$, the optimum LR test can be constructed accordingly.

B. Suboptimum Fusion Rules with Known Channel Envelopes

Implementing the optimum LR-based fusion rule using Λ_1 as given in (7) requires complete channel knowledge of all the hops and the composite local performance indices. To relieve the above requirements, we propose two alternatives as the low and high channel SNR approximations to the optimum LR-based fusion rule. Consider the high SNR case first.

At high SNR, i.e., $\sigma^2 \to 0$, it is easy to show that $P_{1k}^{M_k} \approx 1$ and $P_{2k}^{M_k} \approx 0$. Thus, based on (8) and (9), the composite local performance indices $P_{dk}^{(c)}$ and $P_{fk}^{(c)}$ with known channel envelopes for a multi-hop wireless sensor network can be approximated as

$$P_{dk}^{(c)} \approx P_{dk} \qquad (10)$$
$$P_{fk}^{(c)} \approx P_{fk}. \qquad (11)$$

This leads to the following result.

PROPOSITION 1 *In the high SNR case, the log likelihood ratio (LLR) with known channel envelopes for a multi-hop WSN can be approximated as the Chair-Varshney fusion rule*

$$\Lambda_3 \triangleq \log \Lambda_1 \approx \sum_{y_k<0} \log\left(\frac{1-P_{dk}}{1-P_{fk}}\right) + \sum_{y_k>0} \log\left(\frac{P_{dk}}{P_{fk}}\right). \qquad (12)$$

PROOF Based on (7), (10) and (11), the LR can be approximated as

$$\Lambda_1 \approx \prod_{k=1} \frac{P_{dk} + (1-P_{dk}) e^{-(2y_k h_k^{M_k}/\sigma^2)}}{P_{fk} + (1-P_{fk}) e^{-(2y_k h_k^{M_k}/\sigma^2)}}. \qquad (13)$$

Thus, based on [17, Proposition 1], we have

$$\log \Lambda_1 \approx \sum_{y_k<0} \log\left(\frac{1-P_{dk}}{1-P_{fk}}\right) + \sum_{y_k>0} \log\left(\frac{P_{dk}}{P_{fk}}\right).$$

Equation (12) is the same as the optimum fusion rule derived in [7]. We notice that the high channel SNR approximation for the multi-hop case is the same as the one derived in [15] for the single hop case. Intuitively, at high SNR, each relay node tends to make the right decision, and thus can be ignored in the fusion rule design.

Next, we give the low SNR approximation of Λ_1. We start with the low SNR approximation for the composite performance indices.

LEMMA 1 *At low SNR, $P_{dk}^{(c)}$ and $P_{fk}^{(c)}$ with known channel envelope for a multi-hop WSN can be approximated as*

$$P_{dk}^{(c)} \approx \frac{1}{2} + \frac{2^{M_k}\left(\prod_{m=0}^{M_k-1} h_k^m\right)}{\left(\sqrt{2\pi}\sigma\right)^{M_k}}\left(P_{dk} - \frac{1}{2}\right) \qquad (14)$$

$$P_{fk}^{(c)} \approx \frac{1}{2} + \frac{2^{M_k}\left(\prod_{m=0}^{M_k-1} h_k^m\right)}{\left(\sqrt{2\pi}\sigma\right)^{M_k}}\left(P_{fk} - \frac{1}{2}\right). \qquad (15)$$

A proof is provided in Appendix B.

Given the low SNR approximation of the composite performance indices, we have the following.

PROPOSITION 2 *In the low SNR case, the LLR with known channel envelopes for a multi-hop WSN can be approximated as*

$$\log \Lambda_1 \approx \sum_{k=1}^{K} (P_{dk} - P_{fk}) \frac{2^{M_k+1}\left(\prod_{m=0}^{M_k} h_k^m\right) y_k}{\left(\sqrt{2\pi}\sigma\right)^{M_k} \sigma^2}. \qquad (16)$$

PROOF At low SNR, $\sigma^2 \to \infty$, $e^{(-2y_k h_k^{M_k}/\sigma^2)} \to 1$ and can be approximated by the first-order Taylor series expansion, i.e., $e^{(-2y_k h_k^{M_k}/\sigma^2)} \approx 1 - (2y_k h_k^{M_k}/\sigma^2)$. Based on (7), the LR is then approximated as, for large σ^2,

$$\Lambda_1 \approx \prod_{k=1}^{K} \frac{P_{dk}^{(c)} + (1-P_{dk}^{(c)})\left(1 - \dfrac{2y_k h_k^{M_k}}{\sigma^2}\right)}{P_{fk}^{(c)} + (1-P_{fk}^{(c)})\left(1 - \dfrac{2y_k h_k^{M_k}}{\sigma^2}\right)} \qquad (17)$$

$$= \prod_{k=1}^{K} \frac{1-(1-P_{dk}^{(c)})\frac{2y_k h_k^{M_k}}{\sigma^2}}{1-(1-P_{fk}^{(c)})\frac{2y_k h_k^{M_k}}{\sigma^2}}. \quad (18)$$

Taking logarithm on both sides of (18), we have

$$\log\Lambda_1 \approx \sum_{k=1}^{K}\left\{\log\left[1-(1-P_{dk}^{(c)})\frac{2y_k h_k^{M_k}}{\sigma^2}\right] - \log\left[1-(1-P_{fk}^{(c)})\frac{2y_k h_k^{M_k}}{\sigma^2}\right]\right\}$$

$$\overset{(a)}{\approx} -\sum_{k=1}(1-P_{dk}^{(c)})\frac{2y_k h_k^{M_k}}{\sigma^2} + \sum_{k=1}(1-P_{fk}^{(c)})\frac{2y_k h_k^{M_k}}{\sigma^2}$$

$$= \sum_{k=1}^{K}(P_{dk}^{(c)}-P_{fk}^{(c)})\frac{2y_k h_k^{M_k}}{\sigma^2}$$

$$= \sum_{k=1}^{K}(P_{dk}-P_{fk})\frac{2^{M_k+1}\left(\prod_{m=0}^{M_k} h_k^m\right)y_k}{\left(\sqrt{2\pi}\sigma\right)^{M_k}\sigma^2}. \quad (19)$$

Here we have used the fact that $\log(1+x)\approx x$ when $x\rightarrow 0$ in (a).

Assuming that all the local performance indices are identical, we can rewrite (19) as follows

$$\log\Lambda_1 \approx \frac{2(P_d-P_f)}{\sigma^2}\sum_{k=1}^{K}\left(\frac{2}{\sqrt{2\pi}\sigma}\right)^{M_k}\left(\prod_{m=0}^{M_k} h_k^m\right)y_k. \quad (20)$$

Neglecting the constant term that does not affect detection performance, we have

$$\log\Lambda_1 \approx \sum_{k=1}^{K}\left(\frac{2}{\sqrt{2\pi}\sigma}\right)^{M_k}\left(\prod_{m=0}^{M_k} h_k^m\right)y_k = \sum_{k=1}^{K} h_k^{(c)}y_k \triangleq \Lambda_4 \quad (21)$$

where the composite channel envelopes $h_k^{(c)} = [\prod_{m=0}^{M_k-1}(\sqrt{(2/\pi)}(h_k^m/\sigma))]h_k^{M_k}$.

Λ_4 is similar to the MRC statistic derived as a low SNR approximation for the single hop case in [15], except that the weighting function is the composite channel envelope $h_k^{(c)}$, which involves the product of weighted SNRs of all the hops except the last hop. Clearly, at low SNR (i.e., σ^2 is large), those sensors with more hops are deemphasized.

V. FUSION RULES WITH KNOWN CHANNEL FADING STATISTICS

In this section, assuming that only channel fading statistics are available, we derive the optimum LR-based fusion rule and its two alternatives. Consider Rayleigh fading channels. For simplicity, assume that all links have identical fading statistics. Denoted by $2\sigma_c^2$, the mean squared value of the fading channel envelope, we have, for $k = 1,2,\ldots,K$; $i = 0,1,\ldots,M_k$,

$$f(h_k^i) = \frac{h_k^i}{\sigma_c^2}\exp\left\{-\frac{(h_k^i)^2}{2\sigma_c^2}\right\}. \quad (22)$$

Throughout this section, this probability density function of the Rayleigh fading channel envelope will be used to derive the fusion rules.

A. The Optimum LR-Based Fusion Rule With Known Channel Fading Statistics

Denote Λ_2 as the LR that corresponds to the case when only channel fading statistics are known. As in Section IVA, $P_{1k}^{M_k}$ and $P_{2k}^{M_k}$ can be recursively determined as in the second part of Appendix A. Then using (8) and (9), we can determine $P_{dk}^{(c)}$ and $P_{fk}^{(c)}$. The following theorem gives the form of Λ_2.

THEOREM 2 *The LR with known channel fading statistics and composite local performance indices for a multi-hop WSN is*

$$\Lambda_2 = \prod_{k=1}^{K}\frac{1+[P_{dk}^{(c)}-Q(ry_k)]\sqrt{2\pi}ry_k e^{((ry_k)^2/2)}}{1+[P_{fk}^{(c)}-Q(ry_k)]\sqrt{2\pi}ry_k e^{((ry_k)^2/2)}} \quad (23)$$

where $r = (\sigma_c/\sigma\sqrt{\sigma_c^2+\sigma^2})$ and $Q(\cdot)$ is the complementary distribution function of a standard Gaussian random variable.

PROOF Similar to the result in [16], we have

$$f(y_k \mid u_k^{M_k}=1,H_1) = \frac{\sigma}{\sqrt{2\pi(\sigma_c^2+\sigma^2)}}e^{-(y_k^2/2\sigma^2)}\left[1+Q(-ry_k)\sqrt{2\pi}ry_k e^{((ry_k)^2/2)}\right] \quad (24)$$

$$f(y_k \mid u_k^{M_k}=-1,H_1) = \frac{\sigma}{\sqrt{2\pi(\sigma_c^2+\sigma^2)}}e^{-(y_k^2/2\sigma^2)}\left[1-Q(ry_k)\sqrt{2\pi}ry_k e^{((ry_k)^2/2)}\right]. \quad (25)$$

Then,

$$f(y_k \mid H_1) = f(y_k \mid u_k^{M_k}=1,H_1)P(u_k^{M_k}=1\mid H_1) + f(y_k \mid u_k^{M_k}=-1,H_1)P(u_k^{M_k}=-1\mid H_1)$$
$$= \frac{\sigma}{\sqrt{2\pi(\sigma_c^2+\sigma^2)}}e^{-(y_k^2/2\sigma^2)} \times\left[1+(P_{dk}^{(c)}-Q(ry_k))\sqrt{2\pi}ry_k e^{((ry_k)^2/2)}\right]. \quad (26)$$

Similarly,

$$f(y_k \mid H_0) = f(y_k \mid u_k^{M_k}=1,H_0)P(u_k^{M_k}=1\mid H_0) + f(y_k \mid u_k^{M_k}=-1,H_0)P(u_k^{M_k}=-1\mid H_0)$$
$$= \frac{\sigma}{\sqrt{2\pi(\sigma_c^2+\sigma^2)}}e^{-(y_k^2/2\sigma^2)} \times\left[1+(P_{fk}^{(c)}-Q(ry_k))\sqrt{2\pi}ry_k e^{((ry_k)^2/2)}\right]. \quad (27)$$

Thus,

$$\Lambda_2 = \frac{f(\mathbf{y} \mid H_1)}{f(\mathbf{y} \mid H_0)} = \prod_{k=1}^{K} \frac{1 + [P_{dk}^{(c)} - Q(ry_k)]\sqrt{2\pi} r y_k e^{((ry_k)^2/2)}}{1 + [P_{fk}^{(c)} - Q(ry_k)]\sqrt{2\pi} r y_k e^{((ry_k)^2/2)}}. \quad (28)$$

We notice that both LR-based fusion rules given in (7) and (23) have similar forms to the ones derived in [15] and [16] for the single hop case. However, for the case with multi-hop transmissions, the composite local performance indices $P_{dk}^{(c)}$ and $P_{fk}^{(c)}$ are functions of local performance indices and channel SNRs corresponding to the relay links.

The optimum fusion rule Λ_2 as in (23) for known channel fading statistics requires knowledge of the composite local performance indices, which involve the number of relays for each local sensor and channel parameters. Next, we derive several suboptimum fusion rules that alleviate the requirement of a priori information.

B. Suboptimum Fusion Rules With Known Channel Fading Statistics

Again, we start by considering the low and high channel SNR approximations of the optimum LR fusion rule Λ_2.

At high SNR, we still have $P_{1k}^{M_k} \approx 1$ and $P_{2k}^{M_k} \approx 0$. Thus, the composite performance indices with known channel fading statistics for a multi-hop WSN can be approximated as

$$P_{dk}^{(c)} \approx P_{dk} \quad (29)$$

$$P_{fk}^{(c)} \approx P_{fk}. \quad (30)$$

This leads to the same approximation as before, i.e., in the high SNR case, the LLR with known channel fading statistics for a multi-hop WSN reduces to the Chair-Varshney fusion rule

$$\log \Lambda_2 \approx \sum_{y_k<0} \log\left(\frac{1-P_{dk}}{1-P_{fk}}\right) + \sum_{y_k>0} \log\left(\frac{P_{dk}}{P_{fk}}\right). \quad (31)$$

To show this, we substitute (29) and (30) into (23), hence the LR can be approximated as

$$\Lambda_2 \approx \prod_{k=1}^{K} \frac{1 + (P_{dk} - Q(ry_k))\sqrt{2\pi} r y_k e^{((ry_k)^2/2)}}{1 + (P_{fk} - Q(ry_k))\sqrt{2\pi} r y_k e^{((ry_k)^2/2)}}. \quad (32)$$

Similar to [16, Proposition 1], we can establish in a straightforward manner that

$$\log \Lambda_2 \approx \sum_{y_k<0} \log\left(\frac{1-P_{dk}}{1-P_{fk}}\right) + \sum_{y_k>0} \log\left(\frac{P_{dk}}{P_{fk}}\right) = \Lambda_3. \quad (33)$$

That is, at high channel SNR, the Chair-Varshney rule approaches the optimum LR-based fusion rule with only the knowledge of the channel fading statistics. This high channel SNR approximation is the same as that in Section IV for the known channel envelopes case.

Next, we give the low SNR approximation of Λ_2. Again, we start with the low SNR approximation of the composite performance indices.

LEMMA 2 *At low SNR, $P_{dk}^{(c)}$ and $P_{fk}^{(c)}$ with known channel fading statistics for a multi-hop WSN can be approximated as*

$$P_{dk}^{(c)} \approx \frac{1}{2} + \left(\frac{\sigma_c}{\sigma}\right)^{M_k}\left(P_{dk} - \frac{1}{2}\right) \quad (34)$$

$$P_{fk}^{(c)} \approx \frac{1}{2} + \left(\frac{\sigma_c}{\sigma}\right)^{M_k}\left(P_{fk} - \frac{1}{2}\right). \quad (35)$$

We give the proof in Appendix C.

Given the low SNR approximation of the composite performance indices, we have the following.

PROPOSITION 3 *At low SNR, the LLR with known channel fading statistics for a multi-hop WSN can be approximated as*

$$\log \Lambda_2 \approx \sum_{k=1}^{K} \sqrt{2\pi} r (P_{dk} - P_{fk}) \left(\frac{\sigma_c}{\sigma}\right)^{M_k} y_k. \quad (36)$$

PROOF To show this, we notice that in the low SNR case, $\sigma^2 \to \infty$, hence $r \to 0$. Therefore,

$$\Lambda_2 \approx \prod_{k=1}^{K} \frac{1 + \left[\frac{1}{2} + \left(\frac{\sigma_c}{\sigma}\right)^{M_k}\left(P_{dk} - \frac{1}{2}\right) - \left(\frac{1}{2} - \frac{ry_k}{\sqrt{2\pi}}\right)\right] \times \sqrt{2\pi} r y_k \left(1 + \frac{r^2 y_k^2}{2}\right)}{1 + \left[\frac{1}{2} + \left(\frac{\sigma_c}{\sigma}\right)^{M_k}\left(P_{fk} - \frac{1}{2}\right) - \left(\frac{1}{2} - \frac{ry_k}{\sqrt{2\pi}}\right)\right] \times \sqrt{2\pi} r y_k \left(1 + \frac{r^2 y_k^2}{2}\right)} \quad (37)$$

$$\approx \prod_{k=1}^{K} \frac{1 + \left[\left(\frac{\sigma_c}{\sigma}\right)^{M_k}\left(P_{dk} - \frac{1}{2}\right) + \frac{ry_k}{\sqrt{2\pi}}\right]\sqrt{2\pi} r y_k}{1 + \left[\left(\frac{\sigma_c}{\sigma}\right)^{M_k}\left(P_{fk} - \frac{1}{2}\right) + \frac{ry_k}{\sqrt{2\pi}}\right]\sqrt{2\pi} r y_k} \quad (38)$$

$$= \prod_{k=1}^{K} \frac{1 + \left(\frac{\sigma_c}{\sigma}\right)^{M_k}\left(P_{dk} - \frac{1}{2}\right)\sqrt{2\pi} r y_k + r^2 y_k^2}{1 + \left(\frac{\sigma_c}{\sigma}\right)^{M_k}\left(P_{fk} - \frac{1}{2}\right)\sqrt{2\pi} r y_k + r^2 y_k^2} \quad (39)$$

where we have used the approximations of $e^{((ry_k)^2/2)} \approx 1 + (r^2 y_k^2/2)$ and $Q(ry_k) \approx (1/2) - (ry_k/\sqrt{2\pi})$ for small r.

Taking logarithm on both sides of (39),

$$\log \Lambda_2 \approx \sum_{k=1}^{K} \log\left[1 + \left(\frac{\sigma_c}{\sigma}\right)^{M_k}\left(P_{dk} - \frac{1}{2}\right)\sqrt{2\pi} r y_k + r^2 y_k^2\right]$$
$$- \sum_{k=1}^{K} \log\left[1 + \left(\frac{\sigma_c}{\sigma}\right)^{M_k}\left(P_{fk} - \frac{1}{2}\right)\sqrt{2\pi} r y_k + r^2 y_k^2\right]$$
$$\overset{a}{\approx} \sum_{k=1}^{K} \left\{\left[\left(\frac{\sigma_c}{\sigma}\right)^{M_k}\left(P_{dk} - \frac{1}{2}\right)\sqrt{2\pi} r y_k + r^2 y_k^2\right]\right.$$
$$\left. - \left[\left(\frac{\sigma_c}{\sigma}\right)^{M_k}\left(P_{fk} - \frac{1}{2}\right)\sqrt{2\pi} r y_k + r^2 y_k^2\right]\right\}$$
$$= \sum_{k=1}^{K} \sqrt{2\pi} r (P_{dk} - P_{fk}) \left(\frac{\sigma_c}{\sigma}\right)^{M_k} y_k \quad (40)$$

where (a) follows from the fact that $\log(1 + x) \approx x$ for small x.

If we assume further that all the local sensors have identical local performance, then

$$\log \Lambda_2 \approx \sqrt{2\pi} r (P_d - P_f) \sum_{k=1}^{K} \left(\frac{\sigma_c}{\sigma}\right)^{M_k} y_k \quad (41)$$

which is equivalent in detection performance to

$$\hat{\Lambda}_5 = \sum_{k=1}^{K} \left(\frac{\sigma_c}{\sigma}\right)^{M_k} y_k. \quad (42)$$

The suboptimum fusion rule $\hat{\Lambda}_5$ again deemphasizes those sensors with more hops in the low SNR regime.

If all the local sensors have the same number of hops, i.e., M_k are the same, one can neglect any constant term that does not affect detection performance. Therefore, $\hat{\Lambda}_5$ is equivalent to

$$\Lambda_5 \triangleq \sum_{k=1}^{K} y_k. \quad (43)$$

Notice that (43) is analogous to the EGC statistic as the constant does not affect detection performance. This EGC form of the low SNR approximation of the fusion rule for the multi-hop case is similar to that obtained in [16] for the single hop case.

VI. SIMULATION RESULTS

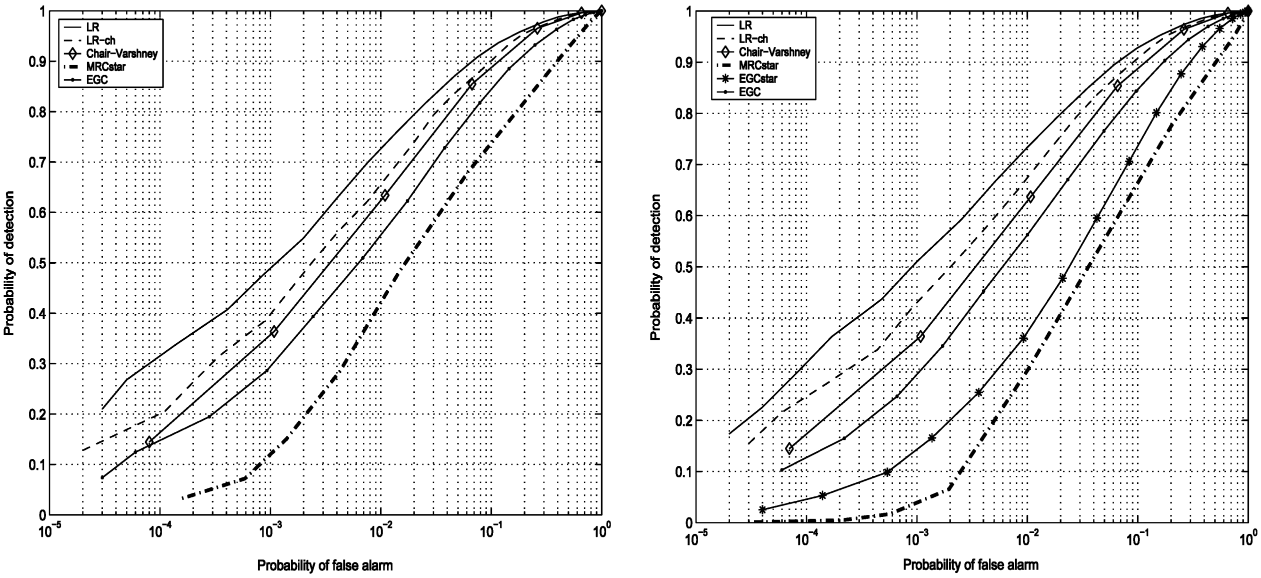
In this section, we compare the performance of the fusion rules proposed in Sections IV and V using simulation. For ease of SNR calculation, we assume that all the channels follow Rayleigh fading with unit mean squared value (unless otherwise specified), i.e., $2\sigma_c^2 = 1$. Binary decisions $u_k^i \in \{+1, -1\}$, $k = 1, 2, \ldots, K$, and $i = 0, 1, \ldots, M_k$, are made at the local sensors and the relay nodes. The channel coefficients h_k^i, for $k = 1, 2, \ldots, K$, and $i = 0, 1, \ldots, M_k$ are generated using the Rayleigh fading model. In all the figures, "*LR*" corresponds to the optimum fusion rule Λ_1 using complete channel knowledge; "*LR – ch*" corresponds to the LR-based fusion rule Λ_2 assuming only the knowledge of the channel statistics; "Chair-Varshney" refers to the statistic Λ_3; "MRCstar" refers to the statistic Λ_4; "EGCstar" refers to the statistic $\hat{\Lambda}_5$; and "EGC" corresponds to the statistic Λ_5. Because $\hat{\Lambda}_5$ is equivalent to Λ_5 when local sensors require the same number of hops to reach the fusion center, we only consider Λ_5 to avoid confusion.

Fig. 3 presents the receiver operating characteristic (ROC) curves at 10 dB channel SNR with 8 local sensors. In Fig. 3(a), we consider the case where local sensors require the same number of hops to reach the fusion center. Specifically, we use the hop number vector [2 2 2 2 2 2 2 2]. In Fig. 3(b), the local sensors require different number of hops to reach the fusion center. In particular, we use the hop number vector [1 1 2 2 2 2 3 3]. The kth element of a hop number vector represents the number of hops required for u_k^0 to reach the fusion center. The ROC curves are obtained using 10^6 Monte Carlo runs. From the figure, it is evident that the optimum LR-based fusion rule (Λ_1) using complete channel knowledge gives the uniformly most powerful detection performance, while the fading statistic based LR fusion rule (Λ_2) gives slightly worse performance than Λ_1. For the particular parameter settings used here, as shown in Figs. 3(a) and (b), the performance of Λ_4 (MRCstar) degrades when the local sensors require unequal number of hops compared with the equal number of hops case. The other fusion statistics behave quite similarly for both equal and unequal number of hops cases. An interesting observation is that, while Λ_5 requires minimum a priori information compared with Λ_4 and $\hat{\Lambda}_5$, it provides a robust detection performance.

To better understand the performance difference as a function of channel SNR, Figs. 4–6 give the probability of detection P_d as a function of channel SNR for various fusion rules and different scenarios. We set a constant system false alarm rate of $P_{f0} = 0.01$. The channel SNR ranges from -10 dB to 20 dB. Each P_d value is obtained over 30000 Monte Carlo trials. Fig. 4 considers the symmetric scenarios with identical local performances and same number of hops among local sensors. Specifically, local $P_{fk} = 0.05$ and $P_{dk} = 0.5$. In Fig. 4(a), with number of sensors $k = 8$, we compare two different cases of number of hops with 2 hops and 10 hops for each local sensor, respectively. In Fig. 4(b), we increase K to 16. We observe the following.

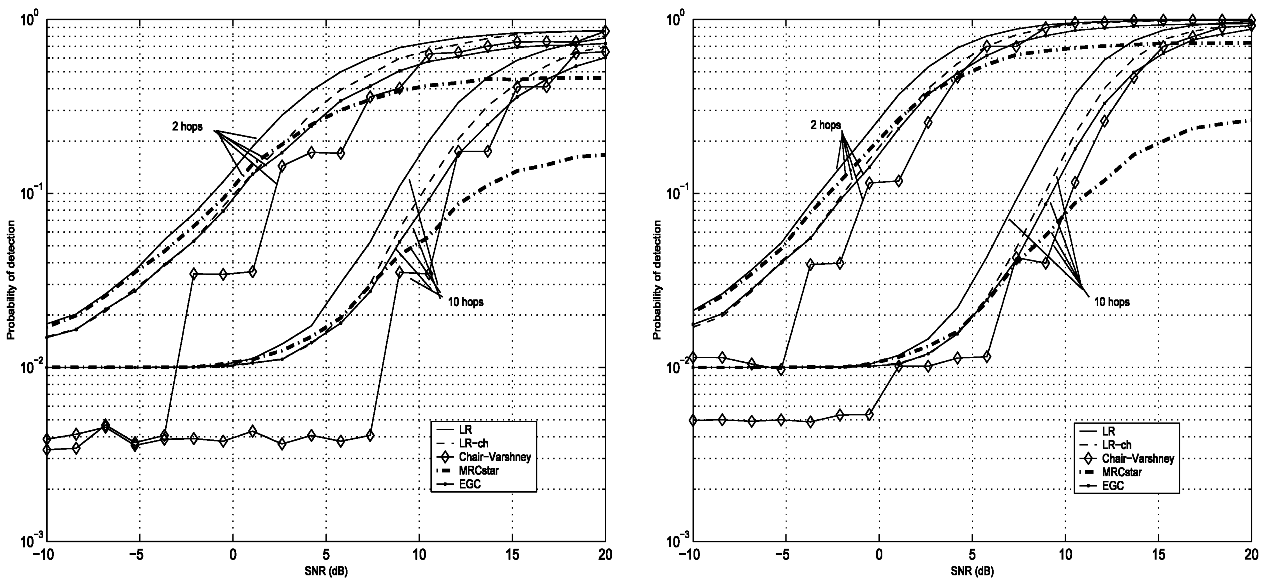
1) At low SNR and high SNR, the MRC-like statistic Λ_4 and the Chair-Varshney rule approach the optimum LR fusion rule Λ_1, respectively; while the statistic Λ_5 and the Chair-Varshney rule approach Λ_2 at low and high SNR, respectively.

2) Each fusion statistic achieves a better performance (larger P_d) with increasing SNR.



(a) (b)

Fig. 3. ROC curves for various fusion rules for Rayleigh fading channels with 8 sensors and SNR = 10 dB. (a) Hop number vector [2 2 2 2 2 2 2 2]. (b) Hop number vector [1 1 2 2 2 2 3 3].



(a) (b)

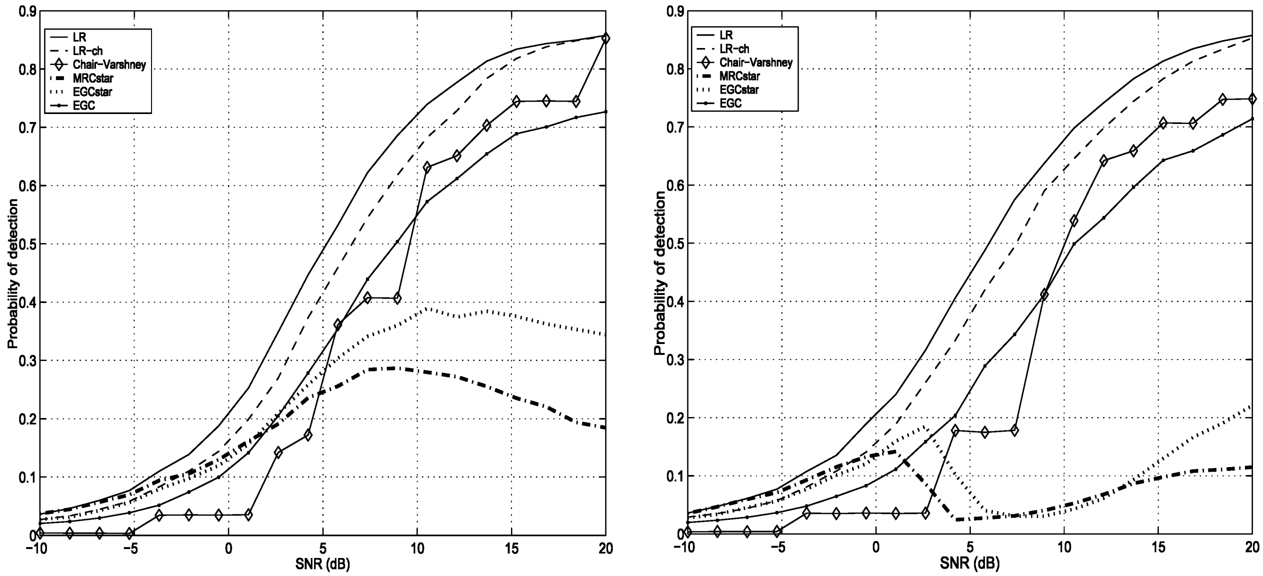
Fig. 4. Probability of detection as function of channel SNR for various fusion rules for Rayleigh fading channels with system level $P_{f0} = 0.01$, local $P_{fk} = 0.05$, $P_{dk} = 0.05$. (a) $K = 8$. (b) $K = 16$.

3) When the number of sensors increases, the performance of each fusion statistic also improves.

4) As expected, a larger number of hops leads to performance degradation.

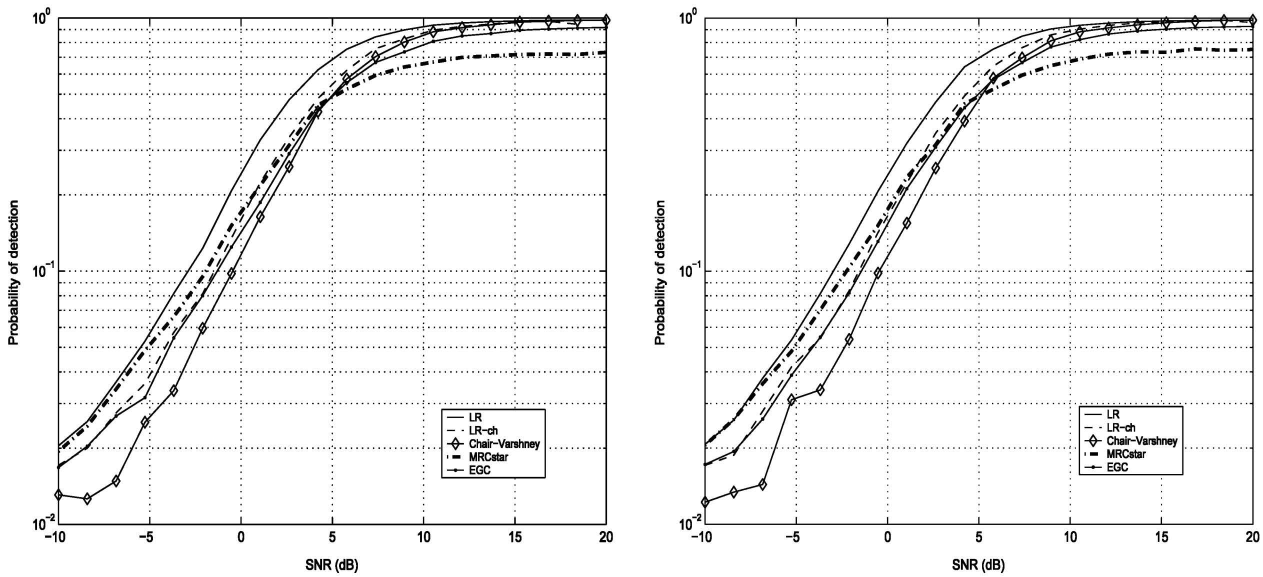
5) There is a stepwise increase associated with the Chair-Varshney approach under the current parameter setting, as shown in Fig. 4(a) and (b). Under the condition of identical local performances and $P_{dk} = 0.5$, the Chair-Varshney statistic is equivalent to a binomial (K, p) distribution with a fixed success probability $p = 0.5$ under hypothesis H_1 [16]. Thus, with a finite number of sensors, even for different SNR values, the system probability of detection still could be the same within a certain range of SNR. A more detailed explanation on this "step" behavior can be found in [16].

Fig. 5 shows how the fusion rules behave when local sensors require unequal number of hops. Specifically, we consider two examples with hop number vector [1 1 2 2 2 2 3 3] in Fig. 5(a) and [1 1 2 2 2 2 8 8] in Fig. 5(b), respectively. Fig. 5(b) corresponds to the case where some local sensors require only a few hops while other local sensors



(a) (b)

Fig. 5. Probability of detection as function of channel SNR for various fusion rules for Rayleigh fading channels with system level $P_{f0} = 0.01$, $K = 8$, local $P_{fk} = 0.05$, $P_{dk} = 0.5$. (a) Hop number vector [1 1 2 2 2 2 3 3]. (b) Hop number vector [1 1 2 2 2 2 8 8].



(a) (b)

Fig. 6. Probability of detection as function of channel SNR for Rayleigh fading channels with system level $P_{f0} = 0.01$, $K = 8$, hop number vector [2 2 2 2 2 2 2 2], local $P_{fk} = 0.05$, local $P_{dk} = [0.5, 0.6, 0.4, 0.8, 0.8, 0.8, 0.6, 0.7]$. (a) With different channel mean square values. (b) With dependent channels.

require a large number of hops to reach the fusion center. The other parameters are set the same as the ones used in Fig. 4(a). In general, there is a performance degradation when the number of hops among local sensors are quite different, as in the case shown in Fig. 5(b). As observed from both Fig. 5(a) and (b), except for the statistics Λ_4 and $\hat{\Lambda}_5$, the other four fusion schemes behave in a similar way as in the equal hop number case. That is, the performance improves as SNR increases. But for the statistics Λ_4 and $\hat{\Lambda}_5$, the performance degrades in a certain SNR range as SNR increases. This unusual behavior is due to the effect of the number of hops and channel SNR dependent weight functions ($((2/\sqrt{2\pi}\sigma)^{M_k}$ in Λ_4 and $(\sigma^c/\sigma)^{M_k}$ in $\hat{\Lambda}_5$, respectively). When SNR is high (σ is small) and the number of hops (M_k) is large, the weight could turn out to be a very large number. Thus, the associated y_k would be given more weight and overemphasized. In other words, the good links (i.e., sensors with less number of hops, thus less weight) would be deemphasized at the fusion stage. This explains why there is a performance degradation in such circumstances. This behavior is also not surprising. Both Λ_4 and $\hat{\Lambda}_5$ are obtained as low SNR approximations of the optimal LR-based fusion rules Λ_1 and Λ_2, respectively. There is no guarantee that they will attain any reasonable performance at high SNR.

In Fig. 6, we illustrate the performance of the various statistics with two asymmetric settings. Specifically, $K = 8$, hop number vector [2 2 2 2 2 2 2 2], local $P_{fk} = 0.05$, but local P_{dk} are different. Specifically, we set $P_{dk} =$ [0.5 0.6 0.4 0.8 0.8 0.8 0.6 0.7] at individual sensors. Fig. 6(a) corresponds to the scenario where the channels have different mean square values. In particular, we randomly generate $2(\sigma^i_{ck})^2$ within (0.5,1.5), for $k = 1,2,\ldots,K$, and $i = 0,1,\ldots,M_k$. The resulting curves show that the performance of each fusion statistic maintains a similar trend as seen in Fig. 4(a). An exception is that there is no stepwise increase for the Chair-Varshney statistic since it is not equivalent to a binomial distribution in general when local P_{dk} take different values.

In many cases, two or more local sensors may share a common relay node in their respective paths to the fusion center. Under this circumstance, the corresponding relay channels may not be independent of each other. Consequently, the conditional independence assumption of y_k is no longer valid. It is therefore of interest to study how the presence of channel dependence affects the detection performance. An example is given in Fig. 6(b) with 8 local sensors and each sensor require 2 hops to reach the fusion center. We assume that sensors 1 and 2 share a common relay node therefore their respective relay links are assumed to share identical channels, i.e., $h^1_1 = h^1_2$. From the plot, the detection performance is quite similar to that of the independent channel case shown in Fig. 6(a).

A comparison of the computation times for obtaining the curves of P_d versus channel SNR for various fusion rules is given in Table I. These simulations were implemented on a 2.40 GHz Intel Pentium(R) 4 processor using MATLAB 6.5 with system false alarm rate $P_{f0} = 0.01$, local performance indices $P_{fk} = 0.05$, $P_{dk} = 0.5$, SNR ranges from -10 dB to 20 dB, but with 30 sample points. Each P_d value is again obtained over 30000 Monte Carlo trials. As expected, as the number of sensors and hops increase, the computation times for simulations also increase. Roughly, the computation time is proportional to the number of sensors.

TABLE I
Computation Time (in Seconds) to Obtain Curves of P_d Versus Channel SNR

Number of Sensors	Hop Number Vector	Hop Number Vector	Hop Number Vector
	[2 2…2]	[6 6…6]	[10 10…10]
$K = 8$	80.406	97.375	124.359
$K = 16$	161.844	198.688	257.125

VII. CONCLUSIONS

In this paper, we have presented a theoretical formulation of the multi-hop decision fusion problem and designed fusion rules for binary decisions transmitted over multi-hop wireless channels undergoing Rayleigh fading in the presence of additive Gaussian noise. We derived the optimum LR-based fusion rule for two cases: with complete channel knowledge and with the knowledge of channel fading statistics. For both cases we showed that the Chair-Varshney fusion rule approaches the optimum LR-based fusion rule at high channel SNR, while at low channel SNR the two LR-based fusion rules reduce to different forms of weighted sums of the fading channel outputs. Both low SNR suboptimum fusion rules deemphasize the sensors with more hops. Specifically, with complete channel knowledge, low channel SNR approximation leads to an MRC-like scheme and the weights are functions of the product of all the link SNRs along each relay path. With known channel fading statistics, the weight involves local performance indices and channel parameters. Under certain conditions, this low channel SNR approximation reduce to a simple EGC form. For most resource-constrained (in both energy and bandwidth) WSN, fusion rules that do not rely on channel coefficients would be practical. Among them, the scheme with an EGC form would be an attractive choice, requiring the least amount of prior information.

We point out here that the conditional independence assumption of y_k may not be valid when the same relay node is used by different sensors and channel fading is really slow. However, our simulation results indicate that under the conditional dependence condition, the derived fusion rules still behave similar to the conditional independence case.

Our work is based on the assumption that the target is not directly observed by the relay nodes. Each relay node makes a simple binary decision based on its noisy input and sends it to the next relay node. This may not be the optimum relay strategy. Further research will focus on the signaling scheme design for the relay node and how the signaling will affect the fusion rule.

APPENDIX A. DERIVATION OF $P^{M_k}_{1k}$ AND $P^{M_k}_{2k}$

Define

$$P^i_k = P(u^i_k = 1 \mid u^{i-1}_k = 1) \tag{44}$$

$$P^i_{1k} = P(u^i_k = 1 \mid u^0_k = 1) \tag{45}$$

$$Q_k^i = P(u_k^i = 1 \mid u_k^{i-1} = -1) \quad (46)$$

$$P_{2k}^i = P(u_k^i = 1 \mid u_k^0 = -1). \quad (47)$$

1) Given the assumption of known channel envelopes, $P_{1k}^{M_k}$ can be recursively determined as follows

$$P_{1k}^1 = P(h_k^0 + n_k^0 > 0) = 1 - Q\left(\frac{h_k^0}{\sigma}\right) \quad (48)$$

$$P_k^i = P(h_k^{i-1} + n_k^{i-1} > 0) = 1 - Q\left(\frac{h_k^{i-1}}{\sigma}\right) \quad (49)$$

$$P_{1k}^{m+1} = P_k^{m+1} P_{1k}^m + (1 - P_k^{m+1})(1 - P_{1k}^m) \quad (50)$$

$$P_{1k}^{M_k} = P_k^{M_k} P_{1k}^{M_k - 1} + (1 - P_k^{M_k})(1 - P_{1k}^{M_k - 1}). \quad (51)$$

$P_{2k}^{M_k}$ can be similarly recursively determined. In fact, because each hop can be viewed as a binary symmetric channel (BSC), we can show that $P_{2k}^{M_k} = 1 - P_{1k}^{M_k}$. The proof is as follows

$$P_{2k}^1 = P(n_k^0 - h_k^0 > 0) = Q\left(\frac{h_k^0}{\sigma}\right) = 1 - P_{1k}^1 \quad (52)$$

$$Q_k^i = P(n_k^{i-1} - h_k^{i-1} > 0) = Q\left(\frac{h_k^{i-1}}{\sigma}\right) = 1 - P_k^i \quad (53)$$

$$P_{2k}^{m+1} = (1 - Q_k^{m+1}) P_{2k}^m + Q_k^{m+1}(1 - P_{2k}^m). \quad (54)$$

Here Q_k^i is equivalent to the crossover probability of the BSC.

Using induction, assume

$$P_{2k}^m = 1 - P_{1k}^m. \quad (55)$$

Then, based on (53), (54), and (55), we have

$$P_{2k}^{m+1} = (1 - P_{1k}^m) P_k^{m+1} + P_{1k}^m (1 - P_k^{m+1}) = 1 - P_{1k}^{m+1}. \quad (56)$$

Thus,

$$P_{2k}^{M_n} = 1 - P_{1k}^{M_n}. \quad (57)$$

2) Given the assumption of known channel fading statistics, we have

$$P_{1k}^1 = \frac{1}{2} + \frac{\sigma_c}{2\sqrt{\sigma_c^2 + \sigma^2}} \quad (58)$$

$$P_k^i = \frac{1}{2} + \frac{\sigma_c}{2\sqrt{\sigma_c^2 + \sigma^2}}. \quad (59)$$

Equations (58) and (59) can be easily derived based on [16, Lemma 1]. Then, by (50) and (51), we can recursively determine $P_{1k}^{M_k}$.

$P_{2k}^{M_k}$ can be obtained in a similar fashion. Alternatively, we still have $P_{2k}^{M_k} = 1 - P_{1k}^{M_k}$.

APPENDIX B. PROOF OF LEMMA 1

In the low channel SNR case, $\sigma^2 \to \infty$, we use the approximation of $Q(h/\sigma) \approx (1/2) - (h/\sqrt{2\pi}\sigma)$, then

$$P_{1k}^1 = 1 - Q\left(\frac{h_k^0}{\sigma}\right) \approx \frac{1}{2} + \frac{h_k^0}{\sqrt{2\pi}\sigma}$$

$$P_{1k}^2 = \left(1 - Q\left(\frac{h_k^0}{\sigma}\right)\right)\left(1 - Q\left(\frac{h_k^1}{\sigma}\right)\right) + Q\left(\frac{h_k^0}{\sigma}\right) Q\left(\frac{h_k^1}{\sigma}\right)$$

$$\approx \frac{1}{2} + \frac{2h_k^0 h_k^1}{\left(\sqrt{2\pi}\sigma\right)^2}$$

$$P_{1k}^k = P_1^{k-1}\left(1 - Q\left(\frac{h_k^{k-1}}{\sigma}\right)\right) + (1 - P_1^{k-1}) Q\left(\frac{h_k^{k-1}}{\sigma}\right)$$

$$\approx \frac{1}{2} + \frac{2^{k-1}\left(\prod_{m=0}^{k-1} h_k^m\right)}{\left(\sqrt{2\pi}\sigma\right)^k} \quad (60)$$

$$P_{1k}^{M_k} \approx \frac{1}{2} + \frac{2^{M_k - 1}\left(\prod_{m=0}^{M_k - 1} h_k^m\right)}{\left(\sqrt{2\pi}\sigma\right)^{M_k}}. \quad (61)$$

Here (60) can be proved by induction.

Similarly, we can get

$$P_{2k}^{M_k} \approx \frac{1}{2} - \frac{2^{M_k - 1}\left(\prod_{m=0}^{M_k - 1} h_k^m\right)}{\left(\sqrt{2\pi}\sigma\right)^{M_k}}. \quad (62)$$

Thus by (8) and (9),

$$P_{dk}^{(c)} \approx \frac{1}{2} + \frac{2^{M_k}\left(\prod_{m=0}^{M_k - 1} h_k^m\right)}{\left(\sqrt{2\pi}\sigma\right)^{M_k}}\left(P_{dk} - \frac{1}{2}\right) \quad (63)$$

$$P_{fk}^{(c)} \approx \frac{1}{2} + \frac{2^{M_k}\left(\prod_{m=0}^{M_k - 1} h_k^m\right)}{\left(\sqrt{2\pi}\sigma\right)^{M_k}}\left(P_{fk} - \frac{1}{2}\right) \quad (64)$$

APPENDIX C. PROOF OF LEMMA 2

In the low channel SNR case, $\sigma^2 \to \infty$, then

$$P_{1k}^1 = \frac{1}{2} + \frac{\sigma_c}{2\sqrt{\sigma_c^2 + \sigma^2}} \approx \frac{1}{2} + \frac{\sigma_c}{2\sigma}$$

$$P_k^i = \frac{1}{2} + \frac{\sigma_c}{2\sqrt{\sigma_c^2 + \sigma^2}} \approx \frac{1}{2} + \frac{\sigma_c}{2\sigma}$$

$$P_{1k}^m \approx \frac{1}{2} + \frac{1}{2}\left(\frac{\sigma_c}{\sigma}\right)^m \quad (65)$$

$$P_{1k}^{M_k} \approx \frac{1}{2} + \frac{1}{2}\left(\frac{\sigma_c}{\sigma}\right)^{M_k}. \quad (66)$$

Equation (65) can be derived by induction.

Similarly,

$$P_{2k}^{M_k} \approx \frac{1}{2} - \frac{1}{2}\left(\frac{\sigma_c}{\sigma}\right)^{M_k}. \quad (67)$$

Thus by (8) and (9),

$$P_{dk}^{(c)} \approx \frac{1}{2} + \left(\frac{\sigma_c}{\sigma}\right)^{M_k} \left(P_{dk} - \frac{1}{2}\right) \tag{68}$$

$$P_{fk}^{(c)} \approx \frac{1}{2} + \left(\frac{\sigma_c}{\sigma}\right)^{M_k} \left(P_{fk} - \frac{1}{2}\right). \tag{69}$$

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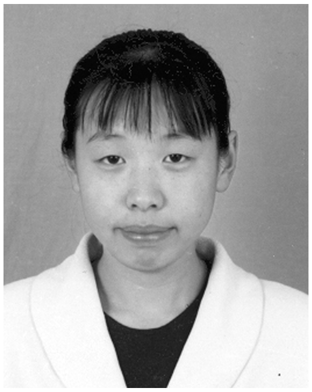

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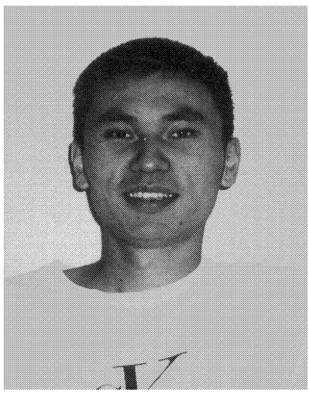

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