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On the Achievable Sum Rate for MIMO Interference Channels

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***Abstract*—In this correspondence, we study some information theoretical characteristics of vector Gaussian interference channels. Resorting to the superposition code technique, a lower bound of the sum capacity for the vector Gaussian interference channel is obtained. Alternatively, orthogonal transmission via frequency division multiplexing is considered and we establish the concavity of sum rate as the bandwidth allocation factor for the vector channel case. Numerical examples indicate that the achievable sum rate via the superposition code compares favorably with orthogonal transmission: the lower bound obtained via the superposition code dominates the best achievable sum rate through orthogonal transmission. This improvement holds for all interference power levels, a sharp contrast to that of the scalar counterpart.**

***Index Terms*—Multiple-input–multiple-output (MIMO) communications, sum capacity, vector Gaussian interference channels.**

I. Introduction

The capacity region of an interference channel (IFC) has been a long standing problem [1]–[4]. Even for the simple scalar Gaussian IFC, capacity region is only known for the very strong interference case [5], [6] and the strong interference case [6]–[8]. An important milestone in IFC is Carleial's work in 1978 [4] where the superposition code idea, originally proposed by Cover for studying the broadcast channels [9], was

Manuscript received October 3, 2005; revised May 25, 2006. This work was supported in part by the National Science Foundation under Grant CCF-0546491, by the Air Force Office of Scientific Research under Grant FA9550-06-1-0051, and by a Syracuse University Graduate Fellowship. The material in this correspondence was presented in part at the IEEE International Symposium on Information Theory, Seattle, WA, July 2006.

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Communicated by R. W. Yeung, Associate Editor for Shannon Theory.
Digital Object Identifier 10.1109/TIT.2006.880051

used to obtain a much improved inner bound for IFC. This inner bound was later refined by Han and Kobayashi [7] who gave an achievable rate region that remains to be the largest reported to date. The advantage of the Han and Kobayashi (HK) bound mainly comes from the simultaneous superposition coding as opposed to sequential superposition coding; the implementation of the original HK bound [7, Theorem 3.1], however, is computationally prohibitive. In [10], Sason rediscovered a subset of the original HK bound using Sato's time sharing idea [11], which is much more amenable to numerical evaluation. Conversely, nontrivial outer bounds have also been obtained for IFC [12]–[14].

It was observed that with moderate interference power, simple orthogonal transmission via frequency or time division multiplexing may outperform the superposition code in terms of achievable sum rate [4]. The intuitive explanation is that the advantage of superposition code and the ensuing successive interference cancellation (SIC) decoding structure is more advantageous with weak or strong interference (i.e., significant power disparity exists between the intended and interfering signals at the receiver). This advantage diminishes when the interference power is comparable to the signal power. We note here that while in theory the original inner bound of Han and Kobayashi [7, Theorem 3.1] includes the achievable region of orthogonal transmissions, its evaluation is numerically prohibitive. There are a number of existing superposition code based achievable regions (all subsets of the original HK bound) that are amenable to numerical evaluation. However, for the moderate interference power case, the achievable sum rates of these known superposition code schemes do not have any advantages over that of orthogonal transmissions [4], [7].[1]

Extending the IFC results to multiple antenna systems has been less well studied. It is straightforward to show that, conditioned on a given set of transmit covariance matrices, many of the classical IFC results directly apply. This, however, is not interesting as the signaling needs to be channel dependent in order to exploit the channel diversity. Vishwanath and Jafar [15] considered some special cases where either the transmitter or the receiver is equipped with a single antenna under the strong or very strong interference assumption. For the general multiple-input–multiple-output (MIMO) systems, obtaining a meaningful achievable capacity region becomes extremely complicated.

In the current work, we consider the achievable sum rate for a two-user vector Gaussian IFC. We present a superposition code approach that improves upon [4] by jointly considering the first two decoding stages therein; this yields an improved sum rate. It turns out that the procedure can be considered as an adaptation of that proposed by Han and Kobayashi [7] (specifically, $\mathcal{G}_0'$ in [7, eq. (5.10), p. 56]) to the vector Gaussian IFC case. We also show that for the vector Gaussian IFC, the achievable sum rate using orthogonal transmission via frequency-division multiplexing (FDM) is a concave function of the bandwidth allocation factor; hence, the maximum achievable sum rate can be easily calculated. We demonstrate via numerical examples that the achievable sum rate of the superposition code approach significantly outperforms that of the orthogonal scheme; and this performance gain holds for all interference power levels. The superiority of interference transmission over orthogonal transmission with MIMO terminals is in contrast to the scalar interference channel case. One can attribute the improved sum rate performance of the superposition code approach largely to the interplay between the spatial diversity and multiuser diversity, as to be elaborated in Section IV.

The rest of this correspondence is organized as follows. In Section II we review the superposition code used by Carleial for computing an inner bound on the capacity region for the scalar Gaussian IFC. The difficulty of applying Carleial's sequential superposition code to the vector channel case motivates the proposed approach that relies on the simultaneous superposition coding idea to obtain a lower bound of the sum capacity. The proposed numerical procedure is described in Section III. In Section IV, we show that the superposition code transmission outperforms orthogonal transmission in terms of achievable sum rate and we conclude in Section V.

[1]A known exception was established in [10, Theorem 3]: when the power constraints are exceedingly generous compared with the noise variance, the achievable sum rate upper bounds that of orthogonal transmissions.

II. A Brief Review of Superposition Coding for Scalar Gaussian IFC

In this section we briefly summarize the superposition code idea for scalar Gaussian IFC. In [4], Carleial showed that the following standard-form Gaussian IFC can be obtained via proper normalization of arbitrary Gaussian IFC:

$$y_1 = x_1 + ax_2 + n_1 \tag{1}$$
$$y_2 = bx_1 + x_2 + n_2 \tag{2}$$

where n_1 and n_2 are independent unit variance Gaussian noises, and x_1 and x_2 are subject to power constraints of P_1 and P_2, respectively.

The very strong interference case corresponds to $a^2 \geq P_1 + 1$ and $b^2 \geq P_2 + 1$ for the Gaussian IFC [5]; or equivalently

$$\begin{aligned} I(x_1; y_2) &\geq I(x_1; y_1|x_2) \\ I(x_2; y_1) &\geq I(x_2; y_2|x_1) \end{aligned} \tag{3}$$

The corresponding capacity region is known to be the rectangle

$$\begin{aligned} R_1 &\leq \frac{1}{2}\log(1+P_1) \\ R_2 &\leq \frac{1}{2}\log(1+P_2) \end{aligned}$$

As such, interference is innocuous to the capacity region as the "very strong interference" can be completely cancelled out through SIC. On the other hand, the strong interference case corresponds to $a^2 \geq 1$ and $b^2 \geq 1$ [6]; or, in general

$$\begin{aligned} I(x_1; y_2|x_2) &\geq I(x_1; y_1|x_2) \\ I(x_2; y_1|x_1) &\geq I(x_2; y_2|x_1). \end{aligned} \tag{4}$$

Its capacity region is known to be

$$\begin{aligned} R_1 &\leq \frac{1}{2}\log(1+P_1) \\ R_2 &\leq \frac{1}{2}\log(1+P_2) \\ R_1 + R_2 &\leq \min\left\{\frac{1}{2}\log(1+P_1+a^2P_2)\right. \\ &\qquad \left.\frac{1}{2}\log(1+b^2P_1+P_2)\right\} \end{aligned}$$

which is effectively the intersection of two multiple-access channels (MAC), each corresponding to one of the two receivers.

The superposition code idea can be most easily motivated by re-examining the strong and very strong interference cases. In both cases, each receiver needs to recover the full interference message as well as its intended message. The difference lies in that the very strong interference case allows each transmitter to communicate at maximum rate while the strong interference case has an added constraint in terms of sum rate. The superposition code proposed in [4] breaks the message into two parts. Instead of trying to decode the entire interference message, each receiver may decode only part of the interference message. Specifically, for the Gaussian interference channel, different combination of successive decoding schemes need to be considered at the receiver and the whole process needs to be repeated for different power allocations [4]. The procedure is briefly summarized below.

TABLE I
FOUR DIFFERENT COMBINATIONS OF DECODERS

	Combination 1	Combination 2	Combination 3	Combination 4
Receiver 1	$x_{10} \to x_{20} \to x_{11}$	$x_{10} \to x_{20} \to x_{11}$	$x_{20} \to x_{10} \to x_{11}$	$\mathbf{x}_{20} \to x_{10} \to x_{11}$
Receiver 2	$x_{20} \to x_{10} \to x_{22}$	$x_{10} \to x_{20} \to x_{22}$	$x_{10} \to x_{20} \to x_{22}$	$x_{20} \to x_{10} \to x_{22}$

Divide each transmitted signal into two components that are independent of each other.

$$x_1 = x_{10} + x_{11}$$
$$x_2 = x_{20} + x_{22}$$

Denote by $\alpha_1 \in [0,1]$ and $\alpha_2 \in [0,1]$ the power allocation factors, i.e.

$$\mathcal{E}\left[x_{10}^2\right] \le \alpha_1 P_1; \quad \mathcal{E}\left[x_{11}^2\right] \le \bar{\alpha}_1 P_1$$
$$\mathcal{E}\left[x_{20}^2\right] \le \alpha_2 P_2; \quad \mathcal{E}\left[x_{22}^2\right] \le \bar{\alpha}_2 P_2$$

where $\bar{\alpha}_i = 1 - \alpha_i$. The splitting of the messages allows partial decoding of the interference signal. Specifically, x_{10}, x_{11}, and x_{20} are to be recovered at receiver 1 while x_{20}, x_{22}, and x_{10} at receiver 2; i.e., x_{10} and x_{20} are "common" information while x_{11} and x_{22} are "private" information. Four different combinations of decoders were considered in [4], as listed in Table I.

For each of the four combinations, one can compute the achievable rate region by varying the power allocation factors. The final inner bound of the capacity region is the union of rate pairs achievable by any of the four combinations. Apparently, the same superposition code procedure can also be used to find a lower bound for the sum capacity, where one simply chooses the maximum sum rate among the four decoders.

III. A LOWER BOUND OF THE SUM CAPACITY FOR VECTOR GAUSSIAN IFC

A. System Model

Consider the following vector Gaussian channel with two mutually interfering transceiver pairs:

$$\mathbf{y}_1 = \mathbf{H}_1\mathbf{x}_1 + \mathbf{H}_{12}\mathbf{x}_2 + \mathbf{n}_1 \tag{5}$$
$$\mathbf{y}_2 = \mathbf{H}_{21}\mathbf{x}_1 + \mathbf{H}_2\mathbf{x}_2 + \mathbf{n}_2 \tag{6}$$

where

- $\mathbf{x}_i$ is a $t_i \times 1$ transmit vector from transmitter i with t_i being the number of elements at transmitter i. The power constraint for $\mathbf{x}_i$ is P_i, i.e.

$$\mathrm{tr}\left(\mathcal{E}\left[\mathbf{x}_i\mathbf{x}_i^T\right]\right) \le P_i$$

- $\mathbf{y}_i$ is a $r_i \times 1$ receive vector at receiver i with r_i being the number of elements at receiver i;
- $\mathbf{H}_i$ is the $r_i \times t_i$ channel matrix corresponding to the ith transceiver pair (intended);
- $\mathbf{H}_{ij}$ is the $r_i \times t_j$ channel matrix between the jth transmitter and ith receiver (interference channel).
- $\mathbf{n}_i$ is a $r_i \times 1$ Gaussian random vectors. Without loss of generality, we assume $\mathbf{n}_i$ has the $r_i \times r_i$ identity matrix $\mathbf{I}_{r_i}$ as its covariance matrix.

This model is illustrated in Fig. 1. In accordance with the discussion of scalar Gaussian interference channels, all variables are assumed to be real. Generalization to complex variables is straightforward and involves only slight modifications.

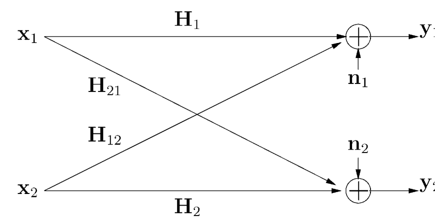

Fig. 1. The MIMO interference channel.

B. A Superposition Coding/Decoding Approach for Vector Gaussian IFC

The superposition coding idea described in Section II does not directly apply to the MIMO IFC. The difficulty lies in the fact that, while for the *scalar* Gaussian IFC an achievable rate region can be determined by power allocation for a given decoding structure, for the *vector* case, the transmitter signaling (in terms of covariance matrices) adds considerable complexity. Second, there is inherent limitation in the decoding combinations as listed in Table I. This limitation is especially pronounced in the vector channel case. For a fixed power allocation, covariance matrices need to be jointly optimized to find the best achievable sum rate. The coupled rate constraints between the two receivers make it difficult to maximize the sum rate for any one of the four decoding combinations in Table I. This difficulty will be further elaborated later. In the following, we describe the decoding scheme that jointly considers the first two decoding stages, thereby improves upon the sequential decoding scheme in [4].

Define the sum capacity as follows.

$$C = \max_{\mathrm{tr}(\mathbf{S}_1) \le P_1, \mathrm{tr}(\mathbf{S}_2) \le P_2} \{R_1 + R_2\}$$

where R_1 and R_2 are the transmit rates such that $\mathbf{x}_i$ can be reliably decoded at receivers i for $i = 1, 2$. Divide each transmitted signal into two components that are independent of each other

$$\mathbf{x}_1 = \mathbf{x}_{10} + \mathbf{x}_{11}$$
$$\mathbf{x}_2 = \mathbf{x}_{20} + \mathbf{x}_{22}$$

The corresponding covariance matrices are

$$\mathbf{S}_1 = \mathbf{S}_{10} + \mathbf{S}_{11}$$
$$\mathbf{S}_2 = \mathbf{S}_{20} + \mathbf{S}_{22}$$

Denote by $\alpha_1 \in [0,1]$ and $\alpha_2 \in [0,1]$ the power allocation factors, i.e.

$$\mathrm{tr}(\mathbf{S}_{10}) \le \alpha_1 P_1; \quad \mathrm{tr}(\mathbf{S}_{11}) \le \bar{\alpha}_1 P_1$$
$$\mathrm{tr}(\mathbf{S}_{20}) \le \alpha_2 P_2; \quad \mathrm{tr}(\mathbf{S}_{22}) \le \bar{\alpha}_2 P_2$$

The proposed decoding scheme can be summarized as a two-stage receiver:

- Receiver 1: $(\mathbf{x}_{10}, \mathbf{x}_{20}) \to \mathbf{x}_{11}$.
- Receiver 2: $(\mathbf{x}_{10}, \mathbf{x}_{20}) \to \mathbf{x}_{22}$.

where the parentheses indicate that the rates for $\mathbf{x}_{10}$ and $\mathbf{x}_{20}$ and the associated covariance matrices are jointly determined. That is, both receivers jointly decode $\mathbf{x}_{10}$ and $\mathbf{x}_{20}$ at the first stage, while $\mathbf{x}_{11}$ and

$\mathbf{x}_{22}$ are separately decoded at the second stage at each individual receiver. This enables joint optimization of the covariance matrices for $\mathbf{x}_{10}$ and $\mathbf{x}_{20}$ instead of obtaining them in a sequential manner. This is feasible due to the following key observation. Upon close inspection of Table I, the first two decoding stages invariably decode $(\mathbf{x}_{10}, \mathbf{x}_{20})$, *albeit* with different decoding orders for different combinations. Consequently, $(\mathbf{x}_{10}, \mathbf{x}_{20})$ can be considered as transmitted signals for two MACs, each corresponding to one of the two receivers. Therefore, the maximum sum rate for $(\mathbf{x}_{10}, \mathbf{x}_{20})$ can be obtained from intersection of the capacity regions for the two MACs.

The advantages over Carleial's decoding scheme are twofold. First, the achievable sum rate for $(\mathbf{x}_{10}, \mathbf{x}_{20})$ using the proposed method clearly upper bounds the sequential decoding combinations from Table I. This is true even for a scalar Gaussian IFC, which is precisely the reason that simultaneous superposition coding is better than sequential superposition coding. For a single Gaussian MAC, the rate region (hence the sum rate) can be obtained by exhausting different decoding orders using SIC. However, for the intersection of two different MACs, the four different combinations of decoding orders are not sufficient. Second, and perhaps more importantly, for the vector case, treating the decoding of $\mathbf{x}_{10}$ and $\mathbf{x}_{20}$ jointly allows one to consolidate the optimization of the covariance matrices $\mathbf{S}_{10}$ and $\mathbf{S}_{20}$ into a single step. This is possible as we are optimizing a single-letter metric, namely the sum rate. This compares more favorably to the sequential decoding schemes in Table I. To elaborate on this, consider, for example, the first combination in Table I, whose achievable rate region for the common information $\mathbf{x}_{10}$ and $\mathbf{x}_{20}$ for given $\mathbf{S}_{10}$ and $\mathbf{S}_{20}$ is

$$\begin{aligned}R_{10} &\leq \min\left\{\frac{1}{2}\log\left|\mathbf{I}_{r_1} + \mathbf{H}_1\mathbf{S}_{10}\mathbf{H}_1^T\left(\mathbf{Z}_1 + \mathbf{H}_{12}\mathbf{S}_{20}\mathbf{H}_{12}^T\right)^{-1}\right|,\right.\\ &\qquad\left.\frac{1}{2}\log\left|\mathbf{I}_{r_2} + \mathbf{H}_{21}\mathbf{S}_{10}\mathbf{H}_{21}^T\mathbf{Z}_2^{-1}\right|\right\} \triangleq R'_{10}\\ R_{20} &\leq \min\left\{\frac{1}{2}\log\left|\mathbf{I}_{r_2} + \mathbf{H}_2\mathbf{S}_{20}\mathbf{H}_2^T\left(\mathbf{Z}_2 + \mathbf{H}_{21}\mathbf{S}_{10}\mathbf{H}_{21}^T\right)^{-1}\right|,\right.\\ &\qquad\left.\frac{1}{2}\log\left|\mathbf{I}_{r_1} + \mathbf{H}_{12}\mathbf{S}_{20}\mathbf{H}_{12}^T\mathbf{Z}_1^{-1}\right|\right\} \triangleq R'_{20}\end{aligned}$$

where, $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are defined in (8) and (9). The maximum sum rate for this combination is

$$R_{10} + R_{20} \leq \max_{\mathrm{tr}(\mathbf{S}_{10})\leq\alpha_1 P_1, \mathrm{tr}(\mathbf{S}_{20})\leq\alpha_2 P_2}\{R'_{10} + R'_{20}\}.$$

This is not a concave function of either $\mathbf{S}_{10}$ or $\mathbf{S}_{20}$ and it is difficult to find even a local maximum given the coupling effects of $\mathbf{S}_{10}$ and $\mathbf{S}_{20}$ on the two rates R_{10} and R_{20}.

The maximization of the sum rate for the proposed procedure is carried out in two steps. In the first step (corresponding to the last decoding stage), we maximize $R_{11} + R_{22}$ with respect to $\mathbf{S}_{11}$ and $\mathbf{S}_{22}$; the corresponding covariance matrices $\mathbf{S}_{11}$ and $\mathbf{S}_{22}$ are then used in the second step (corresponding to the combined decoding stage for $(\mathbf{x}_{10}, \mathbf{x}_{20})$, where $\mathbf{S}_{10}$ and $\mathbf{S}_{20}$ are optimized jointly to maximize $R_{10} + R_{20}$. The maximum sum rate for a given (α_1, α_2) pair is denoted by $R_{\alpha_1\alpha_2}$. The above process needs to be repeated for each (α_1, α_2) pair; the maximum sum rate among all (α_1, α_2) pairs is chosen as the lower bound for the sum capacity.

We describe in details the procedures of obtaining the achievable sum rate and the associated covariance matrices for the proposed scheme.

1) Maximization of $R_{11} + R_{22}$*:* In the last decoding stage, $\mathbf{x}_{10}$ and $\mathbf{x}_{20}$ have been subtracted, after having been successfully decoded at both receivers. The resulting signals at the two receivers are

$$\begin{aligned}\mathbf{y}'_1 &= \mathbf{H}_1\mathbf{x}_{11} + \mathbf{H}_{12}\mathbf{x}_{22} + \mathbf{n}_1\\ \mathbf{y}'_2 &= \mathbf{H}_2\mathbf{x}_{22} + \mathbf{H}_{21}\mathbf{x}_{11} + \mathbf{n}_2\end{aligned}$$

As both receivers use single user detection by treating the other user's remaining signal as pure interference, the maximum sum rate $R_{11} + R_{21}$ is

$$\begin{aligned}&\max_{\mathrm{tr}(\mathbf{S}_{11})\leq\bar{\alpha}_1 P_1, \mathrm{tr}(\mathbf{S}_{22})\leq\bar{\alpha}_2 P_2}\\ &\times\left\{\frac{1}{2}\log\left|\mathbf{I}_{r_1} + \mathbf{H}_1\mathbf{S}_{11}\mathbf{H}_1^T\left(\mathbf{I}_{r_1} + \mathbf{H}_{12}\mathbf{S}_{22}\mathbf{H}_{12}^T\right)^{-1}\right|\right.\\ &\left.+\frac{1}{2}\log\left|\mathbf{I}_{r_2} + \mathbf{H}_2\mathbf{S}_{22}\mathbf{H}_2^T\left(\mathbf{I}_{r_2} + \mathbf{H}_{21}\mathbf{S}_{11}\mathbf{H}_{21}^T\right)^{-1}\right|\right\}\end{aligned} \tag{7}$$

The above function is not a concave function of either of the two covariance matrices $\mathbf{S}_{11}$ and $\mathbf{S}_{22}$. However, alternate optimization algorithm based on gradient method can be devised to maximize the sum rate. That is, the two covariance matrices are to be optimized alternately by fixing the other. The algorithm is guaranteed to converge at least to a local maximum. Convergence happens due to the fact that the sum rate is upper bounded (by, for example, the sum of capacities for the two interference-free channels with respective channel matrices $\mathbf{H}_1$ and $\mathbf{H}_2$) and the fact that the gradient method is guaranteed to improve the achievable rate after each iteration.

2) Maximization of $R_{10} + R_{20}$*:* For the first decoding stage, we rewrite the received signal as

$$\begin{aligned}\mathbf{y}_1 &= \mathbf{H}_1\mathbf{x}_{10} + \mathbf{H}_{12}\mathbf{x}_{20} + \mathbf{n}'_1\\ \mathbf{y}_2 &= \mathbf{H}_2\mathbf{x}_{20} + \mathbf{H}_{21}\mathbf{x}_{10} + \mathbf{n}'_2\end{aligned}$$

where

$$\begin{aligned}\mathbf{n}'_1 &= \mathbf{H}_1\mathbf{x}_{11} + \mathbf{H}_{12}\mathbf{x}_{22} + \mathbf{n}_1\\ \mathbf{n}'_2 &= \mathbf{H}_2\mathbf{x}_{22} + \mathbf{H}_{21}\mathbf{x}_{11} + \mathbf{n}_2\end{aligned}$$

with respective covariance matrices

$$\mathbf{Z}_1 = \mathbf{H}_1\mathbf{S}_{11}\mathbf{H}_1^T + \mathbf{H}_{12}\mathbf{S}_{22}\mathbf{H}_{12}^T + \mathbf{I}_{r_1} \tag{8}$$

$$\mathbf{Z}_2 = \mathbf{H}_2\mathbf{S}_{22}\mathbf{H}_2^T + \mathbf{H}_{21}\mathbf{S}_{11}\mathbf{H}_{21}^T + \mathbf{I}_{r_2} \tag{9}$$

The covariance matrices $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are assumed known given that $\mathbf{S}_{11}$ and $\mathbf{S}_{22}$ have been computed in the previous step.

Obtaining the sum rate for a single vector Gaussian MAC admits an elegant iterative water filling algorithm [16]. This, however does not apply to the current case: the sum rate to be maximized is confined to the intersection of two capacity regions of MACs. There are cases that neither of the sum capacities of the two MACs belongs to the intersection of the two MAC capacities (cf. Fig. 2(a)). We describe below a numerical procedure in computing the maximum sun rate for $R_{10} + R_{20}$.

It is well known that, conditioned on given $\mathbf{S}_{10}$ and $\mathbf{S}_{20}$, the rate region of a MAC with channel matrices $\mathbf{H}_1$ and $\mathbf{H}_{12}$, denoted by $\mathcal{C}_1(\mathbf{S}_{10}, \mathbf{S}_{20})$, is a pentagon specified by

$$\begin{aligned}R_{10} &\leq \frac{1}{2}\log\left|\mathbf{I}_{r_1} + \mathbf{H}_1\mathbf{S}_{10}\mathbf{H}_1^T\mathbf{Z}_1^{-1}\right|\\ R_{20} &\leq \frac{1}{2}\log\left|\mathbf{I}_{r_1} + \mathbf{H}_{12}\mathbf{S}_{20}\mathbf{H}_{12}^T\mathbf{Z}_1^{-1}\right|\\ R_{10} + R_{20} &\leq \frac{1}{2}\log\left|\mathbf{I}_{r_1} + \left(\mathbf{H}_1\mathbf{S}_{10}\mathbf{H}_1^T + \mathbf{H}_{12}\mathbf{S}_{20}\mathbf{H}_{12}^T\right)\mathbf{Z}_1^{-1}\right|\end{aligned}$$

Similarly, the rate region for the other MAC with the channel matrices $\mathbf{H}_2$ and $\mathbf{H}_{21}$ and conditioned on the same $\mathbf{S}_{10}$ and $\mathbf{S}_{20}$, which we denote by $\mathcal{C}_2(\mathbf{S}_{10}, \mathbf{S}_{20})$, is specified by

$$\begin{aligned}R_{10} &\leq \frac{1}{2}\log\left|\mathbf{I}_{r_2} + \mathbf{H}_{21}\mathbf{S}_{10}\mathbf{H}_{21}^T\mathbf{Z}_2^{-1}\right|\\ R_{20} &\leq \frac{1}{2}\log\left|\mathbf{I}_{r_2} + \mathbf{H}_2\mathbf{S}_{20}\mathbf{H}_2^T\mathbf{Z}_2^{-1}\right|\\ R_{10} + R_{20} &\leq \frac{1}{2}\log\left|\mathbf{I}_{r_2} + \left(\mathbf{H}_{21}\mathbf{S}_{10}\mathbf{H}_{21}^T + \mathbf{H}_2\mathbf{S}_{20}\mathbf{H}_2^T\right)\mathbf{Z}_2^{-1}\right|\end{aligned}$$

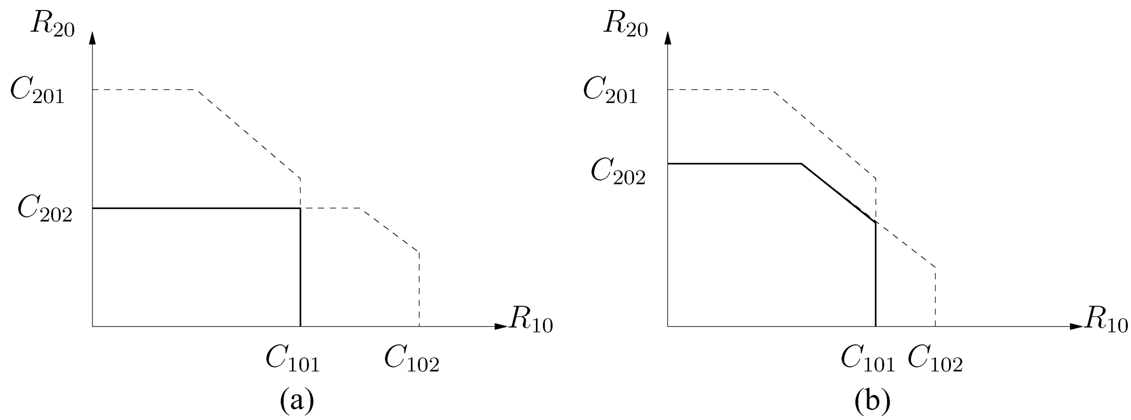

Fig. 2. The achievable regions for (R_{10}, R_{20}), where $C_{101} = \frac{1}{2}\log|\mathbf{I}_{r_1} + \mathbf{H}_1\mathbf{S}_{10}\mathbf{H}_1^T\mathbf{Z}_1^{-1}|$, $C_{102} = \frac{1}{2}\log|\mathbf{I}_{r_2} + \mathbf{H}_{21}\mathbf{S}_{10}\mathbf{H}_{21}^T\mathbf{Z}_2^{-1}|$, $C_{201} = \frac{1}{2}\log|\mathbf{I}_{r_1} + \mathbf{H}_{12}\mathbf{S}_{20}\mathbf{H}_{12}^T\mathbf{Z}_1^{-1}|$, $C_{202} = \frac{1}{2}\log|\mathbf{I}_{r_2} + \mathbf{H}_2\mathbf{S}_{20}\mathbf{H}_2^T\mathbf{Z}_2^{-1}|$.

The achievable rate region for (R_{10}, R_{20}) conditioned on the $(\mathbf{S}_{10}, \mathbf{S}_{20})$ pair is the intersection of the above two regions$\mathcal{C}(\mathbf{S}_{10}, \mathbf{S}_{20}) = \mathcal{C}_1(\mathbf{S}_{10}, \mathbf{S}_{20}) \cap \mathcal{C}_2(\mathbf{S}_{10}, \mathbf{S}_{20})$, specified by

$$R_{10} \le \min\left\{\frac{1}{2}\log\left|\mathbf{I}_{r_1} + \mathbf{H}_1\mathbf{S}_{10}\mathbf{H}_1^T\mathbf{Z}_1^{-1}\right| \right. \\ \left. \frac{1}{2}\log\left|\mathbf{I}_{r_2} + \mathbf{H}_{21}\mathbf{S}_{10}\mathbf{H}_{21}^T\mathbf{Z}_2^{-1}\right|\right\} \triangleq R_{10}^m \tag{10}$$

$$R_{20} \le \min\left\{\frac{1}{2}\log\left|\mathbf{I}_{r_1} + \mathbf{H}_{12}\mathbf{S}_{20}\mathbf{H}_{12}^T\mathbf{Z}_1^{-1}\right| \right. \\ \left. \frac{1}{2}\log\left|\mathbf{I}_{r_2} + \mathbf{H}_2\mathbf{S}_{20}\mathbf{H}_2^T\mathbf{Z}_2^{-1}\right|\right\} \triangleq R_{20}^m \tag{11}$$

$$R_{10} + R_{20} \le \min\left\{\frac{1}{2}\log\left|\mathbf{I}_{r_1} + \left(\mathbf{H}_1\mathbf{S}_{10}\mathbf{H}_1^T + \mathbf{H}_{12}\mathbf{S}_{20}\mathbf{H}_{12}^T\right)\mathbf{Z}_1^{-1}\right| \right. \\ \left. \frac{1}{2}\log\left|\mathbf{I}_{r_2} + \left(\mathbf{H}_{21}\mathbf{S}_{10}\mathbf{H}_{21}^T + \mathbf{H}_2\mathbf{S}_{20}\mathbf{H}_2^T\right)\mathbf{Z}_2^{-1}\right|\right\} \\ \triangleq R_{12}^m. \tag{12}$$

Fig. 2 illustrates two possible cases. For the first case, as in Fig. 2(a), where lines corresponding to the sum capacities for the two individual MACs all reside outside $\mathcal{C}(\mathbf{S}_{10}, \mathbf{S}_{20})$, the maximum sum rate is $R_{10}^m + R_{20}^m$ and in this case $R_{10}^m + R_{20}^m < R_{12}^m$. For the second case, the achievable sum rate of one of the two MACs is contained in $\mathcal{C}(\mathbf{S}_{10}, \mathbf{S}_{20})$, the maximum sum rate is R_{12}^m and in this case $R_{12}^m < R_{10}^m + R_{20}^m$.

Therefore, the maximum sum rate $R_{10} + R_{20}$ for the proposed decoding scheme is

$$\max_{\mathrm{tr}(\mathbf{S}_{10}) \le \alpha_1 P_1, \mathrm{tr}(\mathbf{S}_{20}) \le \alpha_2 P_2} \min\left\{\left(R_{10}^m + R_{20}^m\right), R_{12}^m\right\}. \tag{13}$$

The function to be maximized, $\min\{(R_{10}^m + R_{20}^m), R_{12}^m\}$, is jointly concave in $(\mathbf{S}_{10}, \mathbf{S}_{20})$, its unique maximum can be evaluated fairly straightforwardly. A simple approach is to alternately maximize $\min\{(R_{10}^m + R_{20}^m), R_{12}^m\}$ with respect to one of the covariance matrix by fixing the other. The concavity of the function guarantees that the convergent point is the global maximum.

C. *Orthogonal Transmission*

We now study the maximum sum rate of orthogonal transmission for MIMO IFC. Consider FDM[2] with a bandwidth allocation factor $\beta \in [0, 1]$, i.e., transceiver pair 1 occupies β fraction while transceiver pair 2 occupies $\bar{\beta} = 1 - \beta$ fraction of the total bandwidth. The maximum sum rate for FDM is given by

$$R_1 + R_2 \le \max_{0 \le \beta \le 1} C(\beta)$$

where

$$C(\beta) = \max_{\mathrm{tr}(\mathbf{S}_1) \le P_1, \mathrm{tr}(\mathbf{S}_2) \le P_2} \left\{\frac{\beta}{2}\log\left|\mathbf{I}_{r_1} + \frac{1}{\beta}\mathbf{H}_1\mathbf{S}_1\mathbf{H}_1^T\right| \right. \\ \left. + \frac{\bar{\beta}}{2}\log\left|\mathbf{I}_{r_2} + \frac{1}{\bar{\beta}}\mathbf{H}_2\mathbf{S}_2\mathbf{H}_2^T\right|\right\}. \tag{14}$$

It is obvious that the optimal $\mathbf{S}_1$ and $\mathbf{S}_2$ for each $C(\beta)$ are just water filling covariance matrices. In addition, we show in the Appendix that $C(\beta)$ is a concave function of β. As such, the maximum sum rate for FDM can be computed easily as there is a unique maximum for $C(\beta)$.

IV. Numerical Comparison

For simplicity we assume $t_i = r_i \triangleq n$, for $i = 1, 2$, i.e., all transmitters and receivers are equipped with n antennas. The signal model described in (5) and (6) is amended by introducing two scalar parameters a and b

$$\mathbf{y}_1 = \mathbf{H}_1\mathbf{x}_1 + a\mathbf{H}_{12}\mathbf{x}_2 + \mathbf{n}_1 \tag{15}$$

$$\mathbf{y}_2 = b\mathbf{H}_{21}\mathbf{x}_1 + \mathbf{H}_2\mathbf{x}_2 + \mathbf{n}_2 \tag{16}$$

This enables easy control of the average interference power. In all cases, we assume a unit power constraint for both transmitters, i.e., $\mathrm{tr}(\mathbf{S}_1) = \mathrm{tr}(\mathbf{S}_2) = 1$.

Fig. 3 is a comparison of the two schemes in terms of achievable sum rate as a function of the number of antennas n for $a = b = 1/\sqrt{3}$. The channel matrices are generated whose entries are independent identically distributed unit variance Gaussian random variables. For each antenna number, we compute the sum rate for 20 sets of randomly generated channel matrices and take the average. Apparently, the superposition code transmission enjoys significant advantage over orthogonal transmission as the antenna number increases. Even the obtained *lower bound* for the MIMO IFC has a noticeable margin of improvement over FDM. Notice that for the scalar Gaussian IFC, i.e., (1) and (2), with $a = b = 1/\sqrt{3}$, the sum rate obtained using FDM (or TDM with an average power constraint) outperforms that of the superposition code [4], [7]. Also plotted as a reference is the upper bound obtained by assuming two parallel channels with interference free transmissions. The obtained lower bound is not too far away from this trivial upper bound, indicating the lower bound is reasonably tight.

[2]For the scalar case, FDM and time division multiplexing (TDM) are equivalent in achievable rate region under an average power constraint [4]. It can also be shown that the same holds true for the vector Gaussian IFC case, i.e., the achievable rate regions for FDM and TDM are also identical under an average power constraint.

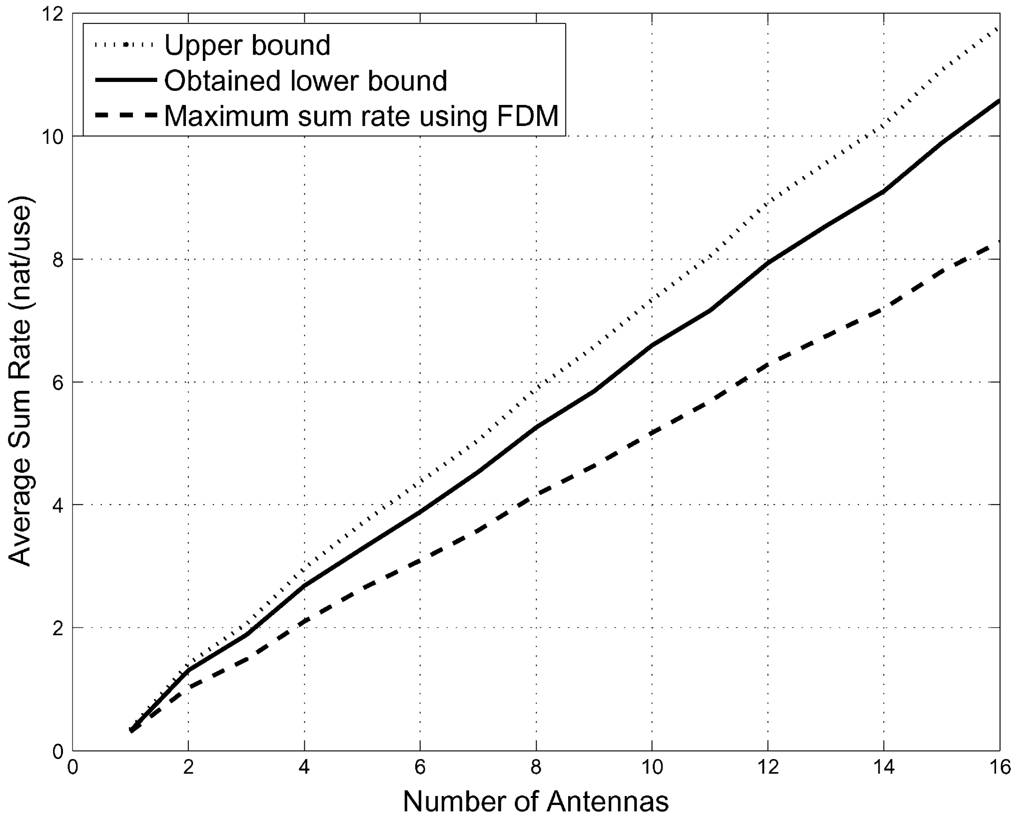

Fig. 3. The average achievable sum rate as a function of antenna number.

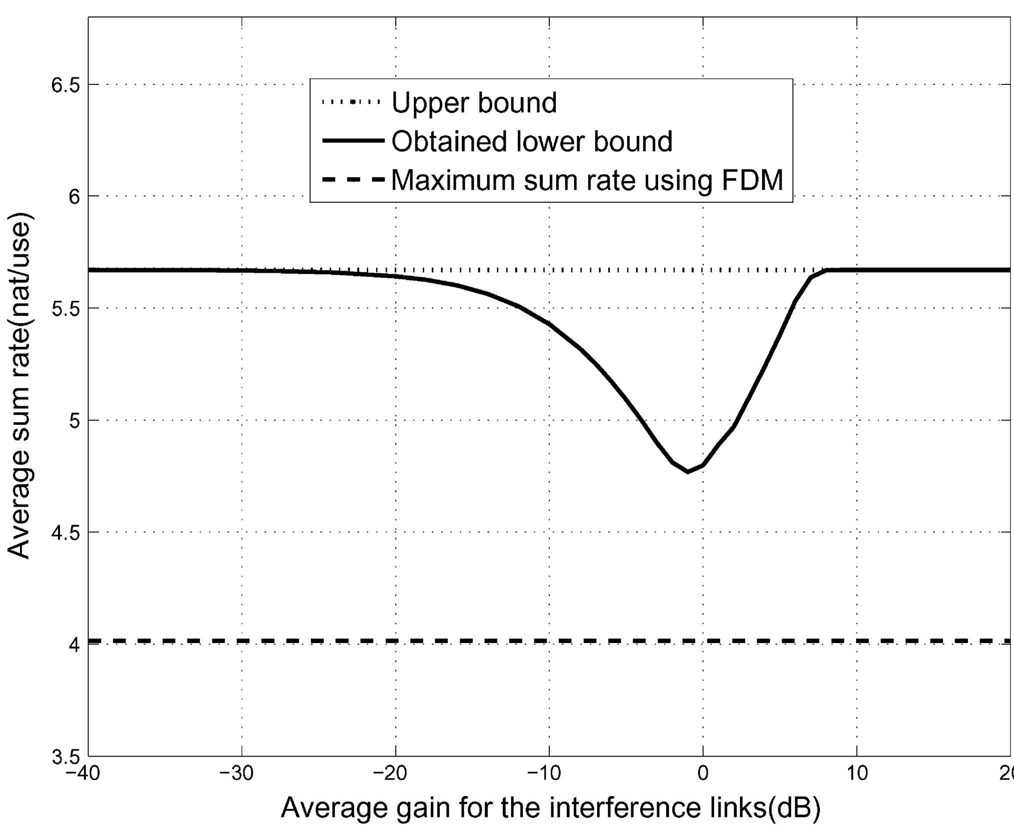

Fig. 4. The achievable sum rate as a function of average interference power for $n = 8$ antennas at both transmitters and receivers.

To understand how the interference power affects the achievable sum rate, we evaluate in Fig. 4 the achievable sum rate as a function of average interference power, defined as $10 \log a^2$ dB with $a = b$. The antenna number is fixed at 8 for both the transmitters and the receivers.

The behavior is similar to that of the scalar case [17] in that both the weak and strong interference cases are rather benign in terms of sum rate as it approaches the interference free case. The sum rate appears to attain its minimum with moderate interference (i.e., when the interference power is of the same magnitude as that of the desired signal). Nonetheless, this lower bound for the interference transmission is consistently better than the sum rate obtained using FDM, a sharp contrast to the scalar Gaussian interference channel case. The upper bound by assuming interference-free transmissions is also plotted; the obtained lower bound is seen to be tight for weak and strong interferences.

The advantage of the superposition code for MIMO IFC over orthogonal transmission is largely attributed to the interplay between multiuser diversity and spatial diversity. For the scalar Gaussian IFC, innocuous interference only occurs if there is significant power disparity between the intended and interfering signals; this is attributed to multiuser diversity in a fading environment with independently faded channels. For the MIMO IFC, the independent channel fading coupled with the presence of multiple antennas make it likely that different channel matrices dwell in nonoverlapping subspace (in terms of its dominant eigenmodes) in addition to possible power disparity. This provides some natural immunity that the transmitter signaling can exploit via appropriate superposition code design. The interplay of spatial and multiuser diversities allows both transceiver pairs to fully utilize the degrees of freedom, as opposed to splitting them in the case of orthogonal transmissions.

V. Conclusion

The sum capacity of MIMO interference channel was studied for the case with channel state information at both transceiver pairs. A procedure to compute a lower bound on the sum capacity was developed using the simultaneous superposition code approach. The achievable sum rate was shown to significantly outperform that of the orthogonal transmission scheme via frequency-division multiplexing for the MIMO channel case, a phenomenon that can be attributed to the interplay between multiuser diversity and spatial diversity.

Appendix
Proof of Concavity of the Maximum Sum Rate for FDM

From (14), we get

$$\begin{aligned}2C(\beta) &= \max_{\mathrm{tr}(\mathbf{S}_1)\le P_1, \mathrm{tr}(\mathbf{S}_2)\le P_2} \Big\{\beta \log \left|\mathbf{I}_{r_1} + \mathbf{H}_1\mathbf{S}_1\mathbf{H}_1^T(\beta\mathbf{I}_{r_1})^{-1}\right| \\ &\quad + \bar{\beta}\log\left|\mathbf{I}_{r_2} + \mathbf{H}_2\mathbf{S}_2\mathbf{H}_2^T(\bar{\beta}\mathbf{I}_{r_2})^{-1}\right|\Big\} \\ &= \max_{\mathrm{tr}(\mathbf{S}_1)\le P_1, \mathrm{tr}(\mathbf{S}_2)\le P_2} \Big\{\beta\log\left|\mathbf{H}_1\mathbf{S}_1\mathbf{H}_1^T + \beta\mathbf{I}_{r_1}\right| \\ &\quad + \bar{\beta}\log\left|\mathbf{H}_2\mathbf{S}_2\mathbf{H}_2^T + \bar{\beta}\mathbf{I}_{r_2}\right| - r_1\beta\log\beta - r_2\bar{\beta}\log\bar{\beta}\Big\}.\end{aligned}$$

Consider the singular value decomposition of $\mathbf{H}_1$ and $\mathbf{H}_2$

$$\begin{aligned}\mathbf{H}_1 &= \mathbf{F}_1\mathbf{\Sigma}_1\mathbf{M}_1^T \\ \mathbf{H}_2 &= \mathbf{F}_2\mathbf{\Sigma}_2\mathbf{M}_2^T.\end{aligned}$$

Define

$$\begin{aligned}\tilde{\mathbf{S}}_1 &= \mathbf{M}_1^T\mathbf{S}_1\mathbf{M}_1 \\ \tilde{\mathbf{S}}_2 &= \mathbf{M}_2^T\mathbf{S}_2\mathbf{M}_2.\end{aligned}$$

We have

$$\begin{aligned}2C(\beta) = \max_{\mathrm{tr}(\tilde{\mathbf{S}}_1)\le P_1, \mathrm{tr}(\tilde{\mathbf{S}}_2)\le P_2} &\Big\{\beta\log\left|\mathbf{\Sigma}_1\tilde{\mathbf{S}}_1\mathbf{\Sigma}_1^T + \beta\mathbf{I}_{r_1}\right| \\ &+ \bar{\beta}\log\left|\mathbf{\Sigma}_2\tilde{\mathbf{S}}_2\mathbf{\Sigma}_2^T + \bar{\beta}\mathbf{I}_{r_2}\right| - r_1\beta\log\beta \\ &- r_2\bar{\beta}\log\bar{\beta}\Big\}\end{aligned}$$

where F_i are dropped since they are unitary matrices.

For a given β, maximizing $C(\beta)$ with respect to $\tilde{\mathbf{S}}_1$ and $\tilde{\mathbf{S}}_2$ are decoupled, hence can be considered independently. Consider $\tilde{\mathbf{S}}_1$ first. From [18], [19], $\tilde{\mathbf{S}}_1$ is the waterfilling matrix for maximum $C(\beta)$. Define the singular values of $\mathbf{H}_1$ as σ_{1i}, $1 \le i \le \min(t_1, r_1)$, with $\sigma_{1i} \ge \sigma_{1,i+1}$, and

$$\tilde{\mathbf{S}}_1 = \mathrm{diag}(\tilde{s}_{11}, \ldots, \tilde{s}_{1t_1})$$

the water filling power allocation for $\tilde{\mathbf{S}}_1$ yields

$$\tilde{s}_{1i} = \left(v - \frac{\beta}{\sigma_{1i}^2}\right)^+$$

where v is chosen such that

$$\sum_{i=1}^{\min(t_1,r_1)} \left(v - \frac{\beta}{\sigma_{1i}^2}\right)^+ = P_1.$$

Assuming, without loss of generality, that $\tilde{s}_{1i} > 0$ for $i = 1, \ldots, n_1$, $n_1 \le \min(t_1, r_1)$, and $\tilde{s}_{1i} = 0$ for $i = n_1 + 1, \ldots, t_1$, i.e., there are n_1 positive $\tilde{s}_{1i}$. We have

$$\begin{aligned}v &= \frac{P_1}{n_1} + \frac{\beta}{n_1}\sum_{i=1}^{n_1}\frac{1}{\sigma_{1i}^2} \\ \tilde{s}_{1i} &= v - \frac{\beta}{\sigma_{1i}^2}; \quad i = 1, 2, \ldots, n_1 \\ \tilde{s}_{1i} &= 0; \quad i = n_1 + 1, \ldots, t_1.\end{aligned}$$

Therefore,

$$\begin{aligned}&\beta\log\left|\mathbf{\Sigma}_1\tilde{\mathbf{S}}_2\mathbf{\Sigma}_1^T + \beta\mathbf{I}_{r_1}\right| \\ &\quad = \beta\log\left\{\left(\prod_{i=1}^{n_1}\sigma_{1i}^2\right)\left(\frac{P_1}{n_1} + \frac{\beta}{n_1}\sum_{i=1}^{n_1}\frac{1}{\sigma_{1i}^2}\right)^{n_1}\beta^{r_1-n_1}\right\}.\end{aligned}$$

Similar waterfilling solutions can be found for the second user. Substitute the results back to $C(\beta)$ and after some simplification, we get

$$\begin{aligned}2C(\beta) &= \beta\log\left(\prod_{i=1}^{n_1}\frac{\sigma_{1i}^2}{n_1}\right) + n_1\beta\log\left(P_1 + \beta\sum_{i=1}^{n_1}\frac{1}{\sigma_{1i}^2}\right) \\ &\quad + \bar{\beta}\log\left(\prod_{i=1}^{n_2}\frac{\sigma_{2i}^2}{n_2}\right) + n_2\bar{\beta}\log\left(P_2 + \bar{\beta}\sum_{i=1}^{n_2}\frac{1}{\sigma_{2i}^2}\right) \\ &\quad - n_1\beta\log\beta - n_2\bar{\beta}\log\bar{\beta}.\end{aligned} \tag{17}$$

Take the second partial derivative with respect to β, we have

$$\begin{aligned}\frac{d^2C(\beta)}{d\beta^2} &= -\frac{n_1P_1^2}{2\left(P_1 + \beta\sum_{i=1}^{n_1}\frac{1}{\sigma_{1i}^2}\right)^2} \\ &\quad - \frac{n_2P_2^2}{2\left(P_2 + \bar{\beta}\sum_{i=1}^{n_2}\frac{1}{\sigma_{2i}^2}\right)^2} < 0.\end{aligned} \tag{18}$$

Therefore, the maximum sum rate is a concave function of β; a unique maximum point exists and can be found in a straightforward manner via nonlinear program.

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On the Reliability Exponents of Two Discrete-Time Timing Channel Models

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***Abstract*—The reliability exponents of two discrete-time single-server timing channel models, considered by Bedekar and Azizoğlu, are determined for rate zero as well as for all rates between the corresponding critical rate and channel capacity. In both models, for rates between zero and the critical rate, we provide random-coding lower bound and straight-line combined with sphere-packing upper bound on the reliability exponent.**

***Index Terms*—Discrete-time queues, reliability exponent, sphere-packing exponent, timing channel.**

I. Introduction

The information capacity of the continuous-time exponential-server queue was determined by Anantharam and Verdú [1], while the reliability exponent of this model has been analyzed by Arikan [5]. Recently, Wagner and Anantharam [7] strengthened Arikan's result by determining the zero-rate exponent of this timing channel model.

In [4], Bedekar and Azizoğlu analyze the information timing capacities of two discrete-time single-server queues. The first model is the discrete-time analogue of the continuous-time exponential-server, and for a queue in which the service time (which is measured in slot units) follows a geometric law the information capacity is determined. In the second queueing model, both multiple arrivals as well as multiple departures are allowed per slot, and for a queue wherein the number of packets the server can handle per slot follows a geometric law the timing capacity is determined.

In this work, we consider the reliability exponents of the above discrete-time queueing models. For both models we derive the random coding and sphere-packing exponents as well as the zero-rate exponent, following similar lines as in [5]–[7]. The first model is treated in Section II, while the model that allows multiple arrivals and multiple departures per slot is treated in Section III.

Notation: We shall henceforth adopt the following notation. Random variables will be denoted by capital letters, while their realizations will be denoted by the respective lower case letters. Whenever the dimension of a random vector is clear from the context, the random vector will be denoted by a bold-face letter, that is, $\boldsymbol{X}$ denotes the random vector $(X_1, X_2, \ldots, X_n)$, and $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ will designate a specific sample value of $\boldsymbol{X}$. However, in those cases where it is important to emphasize explicitly the dimension of a random vector—X^i shall denote the random vector $(X_1, X_2, \ldots, X_i)$, and $x^i = (x_1, x_2, \ldots, x_i)$ will designate a specific sample value of X^i. Furthermore, we write $o_n(1)$ to denote an unspecified positive-valued function that goes to zero as n goes to infinity, and we write $o(n)$ to denote a function such that $o(n)/n = o_n(1)$.

Manuscript received September 1, 2005; revised May 21, 2006. The material in this correspondence was presented in part at the IEEE International Symposium on Information theory. Chicago, IL, June/July 2004.
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Communicated by R. W. Yeung, Associate Editor for Shannon Theory.
Digital Object Identifier 10.1109/TIT.2006.880053