Blind Identification of OFDM Channel Using Receiver Diversity

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Abstract — **In this paper we investigate non data-aided channel estimation for OFDM systems. By exploiting channel diversity with multiple receive antennas, we propose a blind deterministic algorithm that can perfectly recover the channel response using only a single OFDM block. We study the extension of the proposed algorithm to the case when virtual carriers are present and provide some important identifiability results. In addition to being data efficient, the proposed algorithm is independent of the input symbol constellation, computationally efficient, and superior in performance compared with most existing blind algorithms.**

I. Introduction

Because of its immunity to multipath channel fading and spectral efficiency, orthogonal frequency division multiplexing (OFDM) has attracted increasing interest in recent years as a suitable modulation scheme for broadband wireless communication systems, including digital broadcasting and wireless LAN applications.

Coherent OFDM systems require reliable estimation of the time dispersive channel in order to achieve the desired performance gain. Training symbol based OFDM system usually requires extra +20% bandwidth therefore consumes too much precious resources. Blind channel estimation algorithms have the advantages of higher bandwidth efficiency as it does not require the transmission of . Many existing blind OFDM channel estimation methods are statistical in nature (e.g., second order statistics based as in [1, 2, 3, 4, 5]) and usually require large number of data blocks. Clearly, it has limited applicability in wireless channels involving high mobility (large Doppler spread) as the channel may vary from block to block. Deterministic blind OFDM channel estimation, on the other hand, is usually more data efficient. For example, the finite alphabet property is explored in [6, 7]. The decision directed iterative algorithm was proposed in [6] for joint symbol and channel estimation. The performance however largely depends on the initial point and subject to error propagation effect. The proposed identifiability also heavily hinges upon the signal constellation. For example, for 16QAM, the subcarrier number should be at least 52 times the channel length therefore has limited applicability in practice. In [7], finite alphabet is explicitly exploited to obtain an estimate of $H^J(k)$ where $H(k)$ is the channel frequency response at subcarrier k and J is a number determined by the signal constellation. While estimation of $H^J(k)$ can be achieved using a single block for PSK modulation, multiple blocks are still required for QAM modulation along with some statistical assumption on the input symbol. Further, to resolve the phase ambiguity in obtaining $H(k)$ from $H^J(k)$, optimal minimum distance algorithm [7] requires search of J^N possible channel which is usually prohibitive. Here $J = 4$ for QAM modulation and J equals the constellation size for PSK modulation, while N is the number of subcarriers. Even the suboptimal phased directed algorithm can have substantial complexity for moderate to long channel length and is sensitive to the initial starting point of the iteration.

Receiver diversity is another important resource that can be exploited in OFDM channel estimation. In [8, 9] multiple receive antennas are used for channel estimation for OFDM systems without cyclic prefix (CP). The proposed algorithms are subspace methods and usually require large number of blocks. For OFDM systems with CP, receiver diversity was exploited in [10] where a single OFDM is sufficient to obtain reasonable channel estimation for a wide range of SNR values. Channel identifiability conditions were developed in [10] which guarantee perfect channel retrieval in the absence of noise using a single OFDM block. Cramer-Rao lower bound (CRLB) was derived for performance evaluation and it was found that the proposed algorithm is efficient for large SNR.

The model we use in [10] does not contain virtual carriers. Indeed the identifiability condition developed there does not apply to the case when virtual carriers are present. However, we know that in most wireless OFDM system virtual carriers are inserted for anti-aliasing after the D/A converter at the receiver. For example in both IEEE802.11a and HiperLAN/2, 12 out of 64 carriers are virtual carriers. In the paper we extend the channel identification algorithm to the cases with virtual carriers. We will develop in parallel the identifiability conditions for the virtual carriers present case.

The organization of the paper is as follows. In the next section we briefly review our previous work for blind channel estimation using receiver diversity based on non-virtual carrier OFDM model [10]. In section section III, we develop the blind channel estimation algorithm using virtual carriers present OFDM model, along with a new set of identifiability conditions. Simulation results are given in section IV.

The following notations are frequently used in this paper. The DFT matrix $\mathbf{W}$ can be partitioned as $\mathbf{W} = [\mathbf{W}_L|\mathbf{W}_{N-L}]$ where L is the length of channel impulse response which is assumed known *a priori* in this paper, $\mathbf{W}_L$ is the matrix composed of the first L columns of $\mathbf{W}$. Further we can write

$$\mathbf{W}_L = \begin{bmatrix} \mathbf{u}_1^H \\ \vdots \\ \mathbf{u}_N^H \end{bmatrix} \tag{1}$$

where each $\mathbf{u}_k$ is an L by 1 vector. We use bold face capital letters to denote matrices while bold face small letters to denote vectors.

II. Diversity based OFDM channel estimation

Receiver diversity was exploited in [10] where a deterministic blind channel estimation algorithm was presented that can perfectly retrieve the channels using a single OFDM block in the noiseless case. In the presence of noise, the proposed algorithm is both data efficient and computationally efficient and is independent to the signal constellation. We summarize the previous results below.

A Signal Model

In OFDM systems, N subcarriers are used to modulate information symbols to construct one OFDM symbol. Cyclically extended guard time is inserted to maintain inter-carrier orthogonality in the presence of time-dispersive channel [11]. Assuming two receive antennas are used, the received signals, after timing and carrier frequency synchronization and application of DFT, can be written as

$$\begin{aligned} \mathbf{y}_1 &= \mathbf{H}_1\mathbf{d} + \mathbf{z}_1 \\ \mathbf{y}_2 &= \mathbf{H}_2\mathbf{d} + \mathbf{z}_2 \end{aligned}$$

where $\mathbf{H}_i = diag(\mathbf{h}_i)$ with $\mathbf{h}_i = [H_i(0), \cdots, H_i(N-1)]^T$, $H_i(k)$ is the channel frequency response corresponding to i^{th} channel at subcarrier k, $\mathbf{d} = [d_0, \ldots, d_{N-1}]^T$ is the symbol vector, and $\mathbf{z}_1$ and $\mathbf{z}_2$ are additive white complex Gaussian noises and are uncorrelated with each other. Using simple matrix algebra, we can rewrite the signal model as

$$\begin{aligned} \mathbf{y}_1 &= \mathbf{D}\mathbf{h}_1 + \mathbf{z}_1 = \mathbf{D}\mathbf{W}_L\mathbf{g}_1 + \mathbf{z}_1 \\ \mathbf{y}_2 &= \mathbf{D}\mathbf{h}_2 + \mathbf{z}_2 = \mathbf{D}\mathbf{W}_L\mathbf{g}_2 + \mathbf{z}_2 \end{aligned} \qquad (2)$$

where $\mathbf{D} = diag(\mathbf{d})$, and $\mathbf{g}_i$ is the impulse response for the i^{th} channel and is of length L.

B Blind Channel Estimation Algorithm

Consider first the model (2) without noise. It is easy to show by cross multiplication element by element that the following is true

$$y_1(k)\mathbf{u}_k^H\mathbf{g}_2 = y_2(k)\mathbf{u}_k^H\mathbf{g}_1$$

The matrix form of the above equation is

$$\mathbf{Y}_1\mathbf{W}_L\mathbf{g}_2 = \mathbf{Y}_2\mathbf{W}_L\mathbf{g}_1$$

where $\mathbf{Y}_1 = diag(\mathbf{y}_1)$ and $\mathbf{Y}_2 = diag(\mathbf{y}_2)$. Equivalently, we have

$$[\mathbf{Y}_2\mathbf{W}_L | -\mathbf{Y}_1\mathbf{W}_L] \begin{bmatrix} \mathbf{g}_1 \\ \mathbf{g}_2 \end{bmatrix} = 0 \qquad (3)$$

Therefore in the noiseless case, the channel can be retrieved up to a scalar ambiguity by simply finding a solution for the above homogeneous equation. Equivalently, the above equation can be expressed in a quadratic form: $\mathbf{g}^H\mathbf{V}^H\mathbf{V}\mathbf{g} = 0$ where $\mathbf{g} = [\mathbf{g}_1, \mathbf{g}_2]^T$ and $\mathbf{V} = [\mathbf{Y}_2\mathbf{W}_L | -\mathbf{Y}_1\mathbf{W}_L]$. This allows the easy extension to the noisy case where instead of finding the exact solution we simply try to minimize the quadratic term under, for example, a unit norm constraint. The solution is the eigenvector corresponding to the smallest eigenvalue of the matrix $\mathbf{V}^H\mathbf{V}$, which is equivalent to finding the right singular vector corresponding to the smallest singular value of $\mathbf{V}$.

C Implementation using mutliple OFDM blocks

If the channel response is quasi-stationary when we can assume that it remains constant during several OFDM blocks, channel estimation can be improved by utilizing multiple OFDM blocks. Assume K blocks are used for channel estimation, it is straightforward to extend the algorithm to the following minimization problem:

$$\min_{\mathbf{g}} \mathbf{g}^H \left[\sum_{k=1}^{K} \mathbf{V}_k^H\mathbf{V}_k\right] \mathbf{g} \qquad s.t. \quad |\mathbf{g}| = 1$$

where $\mathbf{V}_k$ is constructed for each OFDM block. Notice that this extension does not have any substantial increase in complexity — only one eigen-decomposition is required no matter how many blocks are used.

D Identifiability

The channels are said to be identifiable if in the absence of noise, there is a unique solution (up to a scalar ambiguity) that satisfies the signal model (2). In [10], the following identifiability results are obtained.

Theorem 1 *(sufficient condition) The channel impulse responses $\mathbf{g}_1$ and $\mathbf{g}_2$ can be identified up to a scalar factor if*

1. *$H_1(z)$ and $H_2(z)$ do not share common zeros.*
2. *$N \geq 2L - 1$*

Theorem 2 *(necessary condition) If the channel impulse response $\mathbf{g}_1$ and $\mathbf{g}_2$ are identifiable up to a scalar factor, then $N \geq 2L - 1$.*

III. Diversity Based Channel Estimation In the Presence of Virtual Carriers

In practice, virtual carriers, i.e., those subcarriers that are not used to modulate input symbols, are often inserted for anti-aliasing after D/A converter. The results in [10] are derived using the assumption that no virtual carriers are present. In this section, we extend the results including the identifiability condition to the case with virtual carriers. In the following, we assume that M out of N ($N > M$) subcarriers are used to modulate the information symbols, and we assume, without loss of generality, that the virtual carriers correspond to the last $N - M$ subcarriers.

A Channel identification algorithm

The previously proposed algorithm [10] can be easily modified to adapt to the presence of virtual carriers. Redefine the observation vectors $\mathbf{y}_1$ and $\mathbf{y}_2$ by excluding those virtual carrier components, i.e., $\mathbf{y}_i = [y_i(0), \cdots, y_i(M-1)]^T$ for $i = 1, 2$ and construct $\mathbf{Y}_i = diag(\mathbf{y}_i)$, we can verify that virtually the same algorithm can be applied here.

B Identifiability Conditions

In parallel to Section II.D, we have obtained a similar set of idenfiability conditions.

Theorem 3 *(sufficient condition) The channel impulse responses* $\mathbf{g}_1$ *and* $\mathbf{g}_2$ *can be identified up to a scalar factor if the following conditions hold:*

1. $H_1(z)$ *and* $H_2(z)$ *do not share common zeros.*
2. $M \geq 2L-1$

Proof: In the noiseless case, model (2) yields

$$\begin{aligned} y_1(k) &= d_k \cdot \mathbf{u}_k^H \mathbf{g}_1 \\ y_2(k) &= d_k \cdot \mathbf{u}_k^H \mathbf{g}_2 \end{aligned}$$

Assume we have another set of channel responses $\tilde{\mathbf{g}}_1$ and $\tilde{\mathbf{g}}_2$ that also satisfy the same system model, then we have

$$\begin{aligned} d_k \cdot \mathbf{u}_k^H \mathbf{g}_1 &= \tilde{d}_k \cdot \mathbf{u}_k^H \tilde{\mathbf{g}}_1 \\ d_k \cdot \mathbf{u}_k^H \mathbf{g}_2 &= \tilde{d}_k \cdot \mathbf{u}_k^H \tilde{\mathbf{g}}_2 \end{aligned} \quad (4)$$

From this we get, through cross multiplication,

$$d_k\tilde{d}_k\left(\mathbf{u}_k^H\mathbf{g}_1\right)\left(\mathbf{u}_k^H\tilde{\mathbf{g}}_2\right) = d_k\tilde{d}_k\left(\mathbf{u}_k^H\mathbf{g}_2\right)\left(\mathbf{u}_k^H\tilde{\mathbf{g}}_1\right)$$

Consider the non-virtual carriers only, i.e., for k such that $d_k \neq 0$. If $\tilde{d}_k = 0$, then from (4), $\mathbf{g}_1$ and $\mathbf{g}_2$ must share common zero. Thus $\tilde{d}_k \neq 0$ for non-virtual carriers, and we have

$$H_1(k)\tilde{H}_2(k) = \tilde{H}_1(k)H_2(k)$$

for $k = 0, \ldots, M-1$. Notice that $H_i(k)$ and $\tilde{H}_i(k)$ are respectively Z transform sampled at frequency $2\pi k/N$ for impulse response $\mathbf{g}_i$ and $\tilde{\mathbf{g}}_i$. This is equivalent to, for $z = e^{-j2\pi k/N}$,

$$\left[\sum_{n=1}^{L-1} g_1(n)z^{-n}\right]\left[\sum_{n=1}^{L-1} \tilde{g}_2(n)z^{-n}\right] = \left[\sum_{n=1}^{L-1} g_2(n)z^{-n}\right]\left[\sum_{n=1}^{L-1} \tilde{g}_1(n)z^{-n}\right]$$

Expand the products on both sides,

$$\beta_0 + \beta_1 z^{-1} + \ldots + \beta_{2L-1} z^{-2(L-1)} \mid_{z=e^{-j2\pi kn/N}} = 0$$

for $k = 0, \cdots, M-1$ where

$$\beta_i = \left[\sum_{n=0}^{i} g_1(n)\tilde{g}_2(i-n)\right] - \left[\sum_{n=0}^{i} g_2(n)\tilde{g}_1(i-n)\right]$$

In matrix form:

$$\mathbf{Z}_M \boldsymbol{\beta} = 0$$

The rows of $\mathbf{Z}_M$ are the corresponding M rows of $\mathbf{W}_{2L-1}$, where $\mathbf{W}_{2L-1}$ is the first $2L-1$ columns of DFT matrix $\mathbf{W}$, and $\boldsymbol{\beta} = [\beta_0, \beta_1, \ldots \beta_{2L-1}]^T$. If $M \geq 2L-1$, then the vandermonde matrix $\mathbf{Z}_M$ is full column rank. Therefore

$$\boldsymbol{\beta} = 0 \Longrightarrow \sum_{n=0}^{i} g_1(n)\tilde{g}_2(i-n) = \sum_{n=0}^{i} g_2(n)\tilde{g}_1(i-n)$$

The left and right hand side of the above equation correspond to the linear convolution between $\mathbf{g}_1$ and $\mathbf{g}_2'$, $\mathbf{g}_2$ and $\mathbf{g}_1'$, respectively. Thus we have $H_1(z)\tilde{H}_2(z) = \tilde{H}_1(z)H_2(z)$. Therefore $\Re(H_1(z)) \in \Re(\tilde{H}_1(z)) \cup \Re(H_2(z))$ where $\Re(H_1(z))$ is the set of roots of $H_1(z)$[12]. Since the channels do not share any common zero, we must have

$$\Re(H_1(z)) \in \Re(\tilde{H}_1(z)) \quad (5)$$

Since $\mathbf{g}_1$ and $\tilde{\mathbf{g}}_1$ are of the same length, their corresponding z-transform have the same number of roots. Combined with (5), we have

$$H_1(z) = \alpha\tilde{H}_1(z) \Longrightarrow \mathbf{g}_1 = \alpha\tilde{\mathbf{g}}_1$$

Similarly, we can get $\mathbf{g}_2 = \alpha\tilde{\mathbf{g}}_2$. Q.E.D.

Theorem 4 *(necessary condition) If the channel impulse response* $\mathbf{g}_1$ *and* $\mathbf{g}_2$ *are identifiable up to a scalar factor, then* $M \geq 2L-1$.

The proof is similar to that in [10] and we skip the details.

IV. Performance evaluation

In this section, performance evaluation of the proposed algorithm is carried out both analytically (CRLB) and numerically. Through this section, we use 16 subcarriers with a channel length equal to 5. Randomly generated 16-QAM symbols are used as input symbols.

A Cramer-Rao Lower Bound

In [10], we have obtained the CRLB for channel estimation using diversity scheme. In particular, the FIM corresponding to the complex vectors $\mathbf{g}_1$, $\mathbf{g}_2$, and $\mathbf{d}$ is obtained as

$$\mathbf{F}_c = \frac{1}{\sigma^2}\begin{bmatrix} \mathbf{Q}^H\mathbf{Q} & 0 & \mathbf{Q}^H\mathbf{H}_1 \\ 0 & \mathbf{Q}^H\mathbf{Q} & \mathbf{Q}^H\mathbf{H}_2 \\ \mathbf{H}_1^H\mathbf{Q} & \mathbf{H}_2^H\mathbf{Q} & \mathbf{H}_1^H\mathbf{H}_1 + \mathbf{H}_2^H\mathbf{H}_2 \end{bmatrix},$$

where $\mathbf{Q} = \mathbf{D}\mathbf{W}_L$. Because of the scalar ambiguity, $\mathbf{F}_c$ is at least rank one deficient. The CRLB can be computed by striking out one row and column (assuming the corresponding element is known) of $\mathbf{F}_c$ and taking the inverse of the remaining matrix. In the presence of virtual carriers, we can simply verify that the corresponding FIM is similar to $\mathbf{F}_c$ but except that all the rows and columns corresponding to virtual carriers are excluded.

We compare the mean squared error (MSE) of the proposed algorithm in the presence of virtual carriers to the corresponding CRLB. The result is given in Figure 1 for the following channel pair

$$\mathbf{g}_1 = [-.21-.35i, -.37+.53i, -.10+.03i, .01+.62i, .09-.12i]^T$$
$$\mathbf{g}_2 = [.29-.31i, -.06-.63i, .17+.07i, -.03+.60i, -.03+.10i]^T$$

The last three carriers are chosen as virtual carriers. Further, we assume $g_1(0)$ is known to eliminate the scalar ambiguity and we compute CRLB for $\mathbf{g}_2$. Clearly the proposed algorithm is almost efficient for large SNR.

B Comparison with a subspace method

In this section, we compare the MSE performance of the proposed method to the subspace method in [3]. Since the subspace method does not work for virtual carrier present case, we only consider non virtual carrier present case. The scalar ambiguity is eliminated using a different approach. Assume that the channel estimation algorithm yields an estimate for $\mathbf{g}_1$ as $\tilde{\mathbf{g}}_1$. Define $\alpha = \tilde{\mathbf{g}}_1^H\mathbf{g}_1/|\tilde{\mathbf{g}}_1|^2$, we use $\hat{\mathbf{g}}_1 = \alpha\tilde{\mathbf{g}}_1$ as the channel estimate. The reason is that in the simulation we average the performance over 200 randomly generated channels. The approach of assuming a known channel coefficient may ocasionally lead to trouble for *any blind channel estimation algorithm* if the coefficient happens to be of a very small magnitude.

The result is plotted in Figure 2 where 1000 Monte Carlo runs are used. Clearly, diversity based scheme needs only one OFDM data block to obtain performance equivalent to the subspace method using 64 blocks. Thus the new approach is much more appealing for high mobility applications.

V. Conclusions

In this paper, a receiver diversity based blind channel identification algorithm is extended to the case with virtual carriers. Identifiability conditions are derived that guarantee perfect channel retrieval up to a scalar ambiguity in the absence of noise. The most noteworthy advantage of the proposed algorithm is its data efficiency — a single OFDM data block is sufficient for satisfactory performance. Furthermore, the proposed algorithm imposes no restriction on the input symbol constellation. Simulation is conducted to verify its performance advantage over an existing blind algorithm.

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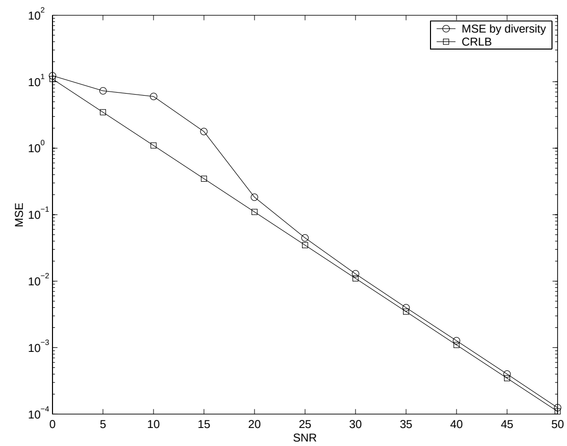

Figure 1: MSE for the blind OFDM channel estimation using diversity scheme and the corresponding CRLB.

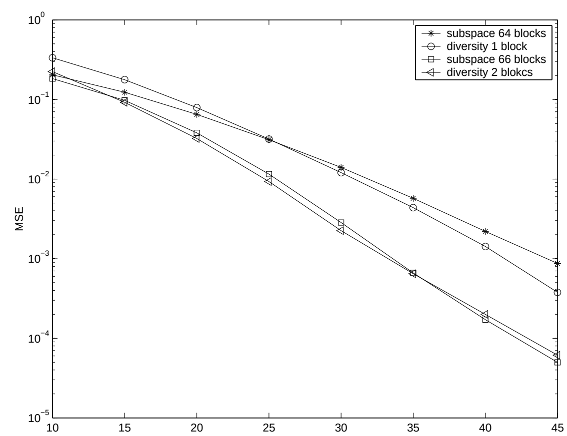

Figure 2: MSE for the blind OFDM channel estimation using diversity scheme and the subspace scheme in [3].