Use of Fuzzy Association Information for Uncertainty Evaluation in Heterogeneous Sensor Networks

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***Abstract*-The paper considers the problem of measurement information fusion from different sources, when one of the sources provides extra information about approximate values of the measured variables or their network associations. Fuzzy models are applied to describe information about the sensor associations and the methods of their application are considered. The properties of the modified estimates are studied in comparison with the conventional ones. The conditions when an extra information application can give a high gain are derived, the gain value is estimated, the practical recommendations are given.**

I. INTRODUCTION

In this paper we investigate the issues of improving reliability and accuracy of the decisions based on an application of the meta-level models (learning or constraint based), which can be deployed in sensor network environments. The key components to develop such a system are:

1. Generation of high-level association models describing the concepts to be learnt. Such models can depend on a variety of factors such as an expert opinion, the choice of the data mining technique employed and the type of data collected from the network sensors. Typically, the process is initiated by collecting the data from the sensor network and pushing it into a database. Subsequently, machine learning techniques like genetic algorithms, neural networks and decision trees can be employed to generate a suitable model between different measurement variables. These models are then reinforced with information integration from non-sensory external sources.
2. Modification of the models generated in the previous step to make them executable in real-time and applicable for joint processing with sensor data.
3. Application of these modified models for sensor validation, verification of the results and reducing the result uncertainty.

One of the current challenges in the sensor network is an advancement of the metrological analysis methods by including new intelligent methodologies. In this group fuzzy and neural methodology is commonly applied alongside with some extra (sometimes a priori) information available [1,2]. In [3] and [1] the object under measurement model which could be presented with neuro-fuzzy methods is applied for a sensor fault detection and even correction. Healy et.al. [3] describe a sensor in-range fault accommodation, which is a fundamental challenge of dual channel control systems in modem aircraft gas turbine engines. An on-board, real-time engine model can be used to provide an analytical third sensor channel that may be used to detect and isolate sensor faults. A fuzzy-logic-based accommodation approach is proposed that enhances the effectiveness of the analytical third channel in the control system's fault isolation and accommodation scheme. In [2] method the number of channels can be expanded and the sensor fault could be corrected. The similar approach [4] is applied for the validation of the measurement results in an ultrasonic sensor dedicated to mobile robot navigation

This paper focuses on the development of a universal methodology, which can be applied for fusion of information of different kinds coming from a variety of sensors on a network including new biological and optical sensor networks, which are researched at the present time.
The goal of this paper is four-fold. It attempts

- to develop the more or less general method allowing fusing the predictions of the values of the measured variables and their linear combinations with the measurement results,
- to evaluate the uncertainty of the modified estimates,
- to evaluate the possible gain, which a prediction application can give us,
- to analyze conditions when the prediction application might produce the strongest benefits.

II. MATHEMATICAL PROBLEM FORMULATION

A conventional way of solving the problem of measurement result estimation assumes its definition as a mathematical programming problem and search for the parameter X' estimates by maximizing some criteria $\hat{X} = \max_x F(Y_1, Y_2, ..., Y_n, X)$,

where F() is a functional, whose shape is determined by the estimation methods,
Yi (i=1,n) is a set of m_i measurement results of the ith variable.

Let us consider extra information as a fuzzy constraint for the parameter vector X and given by the set of membership functions μ(f(X)). The methods of an expert's information acquisition and its propagation through are discussed in [5]. In this case the estimation problem can be considered as an optimization problem with fuzzy constraints. By now research of fuzzy constraints has accumulated different methodologies of solving such problems. One of the simplest and the most obvious way is a unification of both functional criteria and constraints into one synergetic criterion and looking for a global solution as the optimization of such criterion. So the problem can be re-formulated as search for the estimate minimizing the synergetic criterion $\tilde{X} = \max_{X} F(Y1, Y2, \dots, Yn, X) \times \mu(f(X))$

This problem could be tried with conventional or intelligent methods. We will call the solution of this optimization problem a modified estimate and apply it as an estimate of the measured value modified with extra information. The method choice should depend on the estimation techniques applied as well as on the membership function shapes (see [6,7] for more detail).

III. Investigation of the Modified Estimates in Comparison to the Conventional Ones

A. Properties

Let us start our research from the most widely applied in practice the normally distributed measurement results (equation (1)) and the prediction, when the approximate value of a linear combination of a few variables is given (equation (2)),

(1) $Y = AX + \varepsilon y$

(2) $b \approx BX$

where Y is a n×1 vector (under the condition of n > 1) of measurement results,
X is a k×1 vector (under the condition of k > 1) of true values of the measurable variables,
εy is a n×1 vector (under the condition of n > 1) of measurement errors,
b is is a m×1 vector (under the condition of m > 1) of the forecast values,
A, B are matrices giving the structures of measurement and forecast schemes.

(1) could be considered as a standard measurement equation. Let us consider measurement results normally distributed with no bias and the covariation matrix Σy. (2) describes the forecast made that m linear combinations of the measured variables (the combinations are given with the matrix B) approximately have values given by the vector b components. This extra information is described mathematically by using the membership functions (see table 1) with the parameters of fuzziness given with the matrix Σb, which is a diagonal matrix with elements calculated as squares of the forecast fuzziness parameters. With the direct measurements and predictions, matrices A and B become unit matrices and the equations (1) and (2) become simpler as

$Y = X + \varepsilon y$

$b \approx X$

or in a case of one variable it would mean the prediction of an approximate value with the membership function of $\mu(x) = \exp(-(x-b)^2 / \sigma^2)$.

Under general conditions, a conventional maximum likelihood estimate will be calculated as

$\hat{X} = (A^T \Sigma_y^{-1} A)^{-1} A^T \Sigma_y^{-1} Y$;

and a modified estimate should be calculated as

$\tilde{X} = (A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)^{-1} (A^T \Sigma_y^{-1} Y + 2B^T \Sigma_b^{-1} b)$;

or in one variable case

$\hat{X} = Y$; and

$\tilde{X} = (Y/\sigma^2 + 2b/\delta^2)/(1/\sigma^2 + 2/\delta^2) = (Y + b/g^2)/(1 + 2/g^2)$,

where $g = \delta / \sigma$ is the ratio of prediction uncertainty to measurement error, which is called later a prediction uncertainty factor.

The bias and the generalized dispersion of these estimates are equal correspondingly:

$M(\hat{X} - X) = 0$; (3)

$\operatorname{cov}(\hat{X}) = M[(\hat{X} - M\hat{X})(\hat{X} - M\hat{X})^T] = (A^T \Sigma_y^{-1} A)^{-1}$

$M(\tilde{X} - X) = 2(A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)^{-1} B^T \Sigma_b^{-1} (b - BX)$ (4) and $\operatorname{cov}(\tilde{X}) = M[(\tilde{X} - M\tilde{X})(\tilde{X} - M\tilde{X})^T] = (A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)^{-1} A^T \Sigma_y^{-1} A (A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)$

where M() serves as a mean operator.

B. Bias of the modified estimates

Statement 1. The modified estimate coincides with a conventional one if and only if the prediction value coincides with the conventional estimate.

Corollary 1. Modified estimate generally is biased against the conventional one.

Statement 2. Modified estimate lies between the conventional estimate and the forecast value.

Corollary 2. The modified estimate is shifted against the conventional one towards the forecast value.

However, the properties characterizing the accuracy of the modified estimate need to be investigated in order to evaluate a possible gain/lose. The modified estimate becomes unbiased when extra information is absolutely correct prediction or is absent. Actually, the bias of the modified estimate mainly depends on the ratio between the error and the fuzziness of the extra information.

C. Efficiency of the modified estimates

Estimate's accuracy traditionally is taken as its dispersion. Comparing values (3) and (4) one may conclude that when the extra fuzziness is very big (practically no information is given) the dispersions of estimates almost coincide with each other. Generally speaking the modified estimate's dispersion is smaller than the conventional one's, which means that the modified estimate is more efficient than the conventional one. It can be explained by the fact that the modified estimate is "pulled over" towards the forecast value. However, despite this pleasant result the dispersion cannot be taken as a comparison base as the modified estimate could be biased due to the wrong extra information. We have to consider another accuracy indicator, the mean square error (MSE), which is the mean of squares of deviations between the estimate and the true value. This indicator takes into account both the estimate's bias and its dispersion. MSE of the considered estimates will equal to:

$$E_{\hat{X}} = M[(\hat{X} - X)(\hat{X} - X)^T] = (A^T \Sigma_y^{-1} A)^{-1};$$

$$E_{\tilde{X}} = M[(\tilde{X} - X)(\tilde{X} - X)^T] =$$

$$(A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)^{-1} (4B^T \Sigma_b^{-1} (b - BX)(b - BX)^T \Sigma_b^{-1} B$$

$$+ A^T \Sigma_y^{-1} A)(A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)^{-1};$$

The problem of this gain evaluation deserves a special consideration. To evaluate the gain provided by an expert's information application, let us choose the projection of the estimate's MSE, which can be written as

$$T = \frac{1}{K} Sp(E_{\hat{X}} E_{\tilde{X}}^{-1}) =$$

$$= \frac{1}{K} Sp\, [\; (A^T \Sigma_y^{-1} A)^{-1} (A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B) \; (4B^T \Sigma_b^{-1} (b - BX)(b - BX)^T \Sigma_b^{-1} B + A^T \Sigma_y^{-1} A)^{-1}$$

$$(A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)].$$

One may see that in order to increase the maximum gain, the extra information fuzziness should be decreased in comparison with the measurement errors. One may also conclude that if the prediction fuzziness is higher than 100 times measurement errors, the use of extra information becomes doubtful as any possible gain value could be just a few percent. It means that one has to try to decrease the prediction fuzziness parameter. However, this strategy might be risky as it may result in loosing any gain at all.

IV. Is there any sense in use of expert's information and under which conditions?

A. Mathematical point of view

Let us try to clarify conditions when the modified estimate superiors a conventional one against the MSE indicator or becomes more accurate. Mathematically the condition

$E_{\tilde{X}} < E_{\hat{X}}$ can be shown equivalent to the condition

(5) $(b - BX)(b - BX)^T < \Sigma_b + B(A^T \Sigma_y^{-1} A)^{-1} B^T$.

This relationship becomes clearer in the case of direct measurements and predictions when the matrices A and B are unit matrices, and matrices Σ_b and Σy are diagonals. In this case the condition (5) becomes more transparent as

(6) $\sum_{i=1}^{K} (b_i - x_i)^2 / K < \sum_{i=1}^{K} \delta_i^2 / K + \sum_{i=1}^{K} \sigma_i^2 / K$

where K is the number of measured variables,

σi, i=1, K is the root mean square error (RMSE) of the i-th variable measurement errors, and δi, i=1, K is the fuzziness of the ith prediction.

Use of extra information improves accuracy if the mean prediction error is less than the sum of the mean prediction fuzziness and the mean measurement error.

B. Practical point of view

Or in other words what gain could be achieved with a rather inaccurate extra information?

The typical measurement accuracy for the modern measurement instruments could be in the vicinity of 1-2%. In this case, of say 2% measurement error, with the prediction fuzziness of say 20%, which is rather high (for example, it might mean the prediction like "the measured variable has a value of around 10 units or actually somewhere roughly between 8 and 12 units", which in practical cases sounds like a very reasonable suggestion) could achieve the gain up to 44%. However, even in a case when a forecaster makes an error in his/her prediction, there could still be some gain. Speaking very roughly, in order to get any gain the error value should be smaller than a sum of the prediction fuzziness and the measurement error. This allows a forecaster to develop a strategy to avoid loosing. If a forecaster is confident about the prediction value, he/she may low prediction fuzziness and achieve a higher gain. However, when the confidence level decreases, the extra fuzziness could be increased, which might lower the gain value but allow avoiding loses.

V. Conclusion

The problem of extra information use for improvement the measurement procedures quality and the estimates received may be considered as a fusion of information from different sources, which are characterized by different uncertainty degrees. This area attracts a particular attention over a last few years. The problem has been formalized mathematically as an optimization problem with fuzzy constraints and the solution has been found for the normally distributed measurement results and a specific expert's information.

The properties of the modified estimates have been studied in comparison with the conventional ones. The modified estimates have been found more efficient under the condition when the extra information error does not overcome the sum of the average measurement error and the prediction fuzziness. The possible efficiency gain was estimated. The procedures improving reliability of the modified estimates have been offered.

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