ASYMPTOTIC LOCALLY OPTIMAL DETECTOR FOR A LARGE SENSOR NETWORK UNDER THE POISSON REGIME

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ABSTRACT

We consider the distributed detection problem with a large number of identical sensors deployed over a space where the phenomenon of interest has different signal strength depending on location. Each sensor makes a decision based on its own measurement of the spatially varying signal and the local decision of each sensor is sent to a fusion center through a multiple access channel. The fusion center decides whether the phenomenon of interest occurred or not with a global size constraint under Neyman-Pearson context. Assuming that the initial distribution of sensors is a homogeneous spatial Poisson process, we showed that the distribution of alarmed sensors satisfies the locally asymptotic normality (LAN) condition as the number of sensor goes to infinity and derived a new asymptotically locally optimal criterion for the detection of the spatially varying signal. We showed that an optimal test statistic is a weighted sum of local decisions and the optimal weight function is the shape of the spatial signal of interest and the exact value of the spatial signal is not required. It is shown that the spatial signal is translated to the local intensity of alarmed sensors with Poisson assumption and the optimal weight is easily estimated based on the number of alarmed sensors and their locations. For the case of independent, identical distributed (iid) sensor observation, we showed that the counting rule is also asymptotic locally optimal.

Index Terms - Distributed detection, Spatially-varying signal, Poisson distribution, Locally asymptotically normal (LAN), Asymptotically locally optimal, Neyman-Pearson criterion, Fusion rule.

1. INTRODUCTION

Recently large-scale sensor networks draw attention in many applications like environmental monitoring, scientific research, and surveillance. A large number of sensors are expected to be deployed over an area and measure the phenomenon of interest and transmit their local data via wireless channel to a central site which performs a global processing. For these large-scale sensor networks, the number of sensors is expected to be 10,000 to 100,000 and the sensors cover a large area.

The distributed detection using multiple sensors and optimal fusion rules have been widely investigated [7] . Many authors have derived optimal fusion rules based on different level of assumptions [8], [9], [10] and analyzed the asymptotic performance of distributed detection [12], [13]. However, most fusion rules are obtained from the assumption that the hypotheses of the underlying phenomenon are simple, i.e., discrete and M-ary, and they require the knowledge of the false alarm and detection probability of each sensor decision under each hypothesis out of M possible cases. However, for applications like the detection of biological or chemical agents or radioactivity in a certain area, it is difficult to determine the local detection probability at each sensor beforehand and cast to the simple hypotheses testing framework since the signal strength of the phenomenon is not known. In [11], the authors considered the detection of unknown signal via multilevel quantization and simple fusion rules. In addition to the unknown signal strength, for a large-scale sensor network it is reasonable to assume that the phenomenon of interest has different signal strength over area and the observation of each sensor depends on its location and is not identically distributed since sensors are deployed over a large area.

In this paper, we consider the optimal fusion problem under Neyman-Pearson context in such large-scale sensor networks where each sensor observes an unknown spatially-varying signal. Assuming that the initial spatial distribution of sensors is a homogeneous Poisson process and the spatial variation of the signal is known, we derived an asymptotically local optimal detector and an optimal way to utilize the spatial information with the theory of locally asymptotic normality (LAN). We also provide how to estimate the spatial information using the number of alarmed sensors and their locations.

The paper is organized as follows. The data model of the sensor system that we consider is described and a brief summary of Poisson process and locally asymptotic normality (LAN) theory is introduced in section 2. In section 3, the asymptotically locally optimal detector is proposed under the Poisson assumption of sensor distribution and an estimation of spatial information is discussed.

2. SYSTEM MODEL

We consider a large-scale sensor network with identical binary sensors deployed over a field where the phenomenon of interest has spatially-varying strength, and are interested whether the underlying phenomenon of interest has occurred or not based on local decisions of sensors. We denote the spatial signal strength by

$$\gamma(\mathbf{x}) = \theta s(\mathbf{x}), \tag{1}$$

where $\mathbf{x}$ denotes the location, $\theta \in \Theta \stackrel{\Delta}{=} [0,\infty)$ is an unknown amplitude, and $s(\mathbf{x})$ is a known function which incorporates the information about the spatial variation of the underlying phenomenon.

2.1. Single Sensor

We assume that sensors make their local decisions independently without collaborating with other sensors. Since the exact value of the signal strength is unknown, each sensor is designed to detect the following hypotheses

$$\begin{array}{ll} H_0: & \gamma(\mathbf{x}) = 0, \\ H_1: & \gamma(\mathbf{x}) > 0, \end{array} \tag{2}$$

with local size constraint of α_0. The hypotheses (2) is equivalently expressed by

$$\begin{array}{ll} H_0: & \theta = 0, \\ H_1: & \theta > 0. \end{array} \tag{3}$$

The local decision of each sensor S_i located at $\mathbf{x}$ is denoted by

$$u_i = \begin{cases} 0 & \text{if } H_0 \text{ selected}, \\ 1 & \text{otherwise}. \end{cases} \tag{4}$$

One possible sensor observation model is the additive Gaussian noise model where the sensor input Y_i is given by

$$Y_i = \gamma(\mathbf{x}) + N_i, \quad N_i \sim \mathcal{N}(0, \sigma_0), \tag{5}$$

where N_i is the independent sensor input noise. In this case, the local decision rule for (3) at each sensor is the UMP detector given by

$$Y_i \mathop{\gtrless}_{H_0}^{H_1} \tau_0, \tag{6}$$

where $\tau_0 = \sigma_0 Q^{-1}(\alpha_0)$[1]. We define the following probability

$$p(\mathbf{x}) \triangleq \Pr\{u_i = 1\}. \tag{7}$$

Then, $p(\mathbf{x})$ is a function of the signal strength at $\mathbf{x}$ and is given by

$$p(\mathbf{x}) = g(\gamma(x)). \tag{8}$$

For the additive Gaussian observation model, $p(\mathbf{x})$ is expressed as $Q\left(\frac{\tau_0 - \gamma(\mathbf{x})}{\sigma_0}\right)$.

2.2. Parametric Poisson Model

We consider that a large number of sensors in 2.1 are deployed uniformly and randomly over a space A and each sensor makes its decision u_i depending on the signal strength at sensor location. Then, the local decisions are transmitted to the fusion center through multiple access channel where some sensor data can be lost.

We assume that the initial distribution of sensors over field is a homogeneous Poisson process with intensity λ_h. Then, the process of local decision gives a location-dependent thinning of the original sensor distribution with probability $p(\mathbf{x})$ since each sensor decision is independent and based on the signal strength at its location. The data collection through multiple access channel is modeled as an uniform thinning with probability p_m which reflects the data loss during the transmission period through multiple access channel to the collector. The distribution of alarmed sensors, i.e., sensor with $u_i = 1$, is now a nonhomogeneous Poisson process of which the local intensity is given by

$$\lambda(\mathbf{x}) = \lambda_h p_m p(\mathbf{x}) = \lambda_h p_m g(\theta s(\mathbf{x})). \tag{9}$$

[1] $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{1}{2}t^2} dt.$

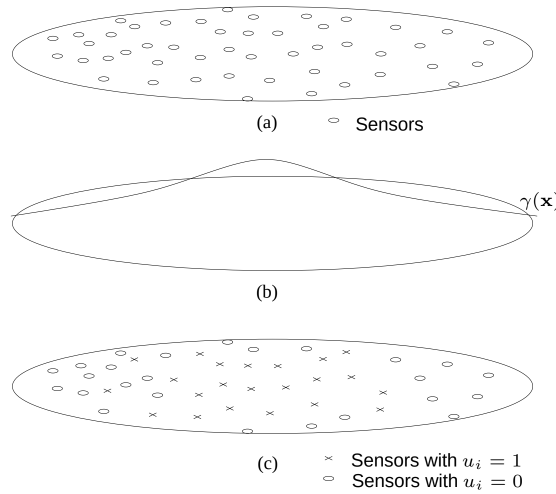

Fig. 1. (a) Initial sensor deployment over area (b) Signal strength of underlying phenomenon (c)Local decisions of sensors

When the function $g(\cdot)$ is linear or θ is in a small neighborhood of $\theta = 0$, the Poisson distribution of alarmed sensors is described by a nonhomogeneous intensity model parameterized by amplitude θ and is given by

$$\lambda(\theta, \mathbf{x}) = \theta f(\mathbf{x}) + \lambda_0, \tag{10}$$

where

$$f(\mathbf{x}) = \lambda_h p_m g'(0) s(\mathbf{x}), \text{ and } \lambda_0 = \lambda_h p_m g(0). \tag{11}$$

The Poisson assumption on the initial sensor distribution effectively changes the global detection to the problem of deciding from which intensity model the spatial distribution of alarmed sensors comes. Notice that the intensity variation $f(\mathbf{x})$ of alarmed sensors is a scaled version of the spatial signal shape $s(\mathbf{x})$.

2.3. Review of Poisson Process

The Poisson distribution X_A in space A is expressed in a simple manner by counting measure notation which is given by

$$X_A(B) = \sum_{i:\ \mathbf{x}_i \in A} \epsilon_{\mathbf{x}_i}(B), \quad \forall B \subset A, \tag{12}$$

where $\mathbf{x}_i, i = 1, \ldots, N_A$ are random points in A, N_A is a Poisson distributed random variable with mean $\Lambda(A)$ and

$$\epsilon_{\mathbf{x}_i}(B) \triangleq \begin{cases} 1, & \mathbf{x}_i \in B \\ 0, & \mathbf{x}_i \notin B \end{cases} \quad \forall B \subset A. \tag{13}$$

We define the stochastic integral for a given function f as

$$I(f) \triangleq \int_A f(\mathbf{x}) X_A(dx) = \sum_{i:\ \mathbf{x}_i \in A} f(\mathbf{x}_i). \tag{14}$$

We denote the probability density of a realization X_A of Poisson distribution with intensity $\lambda(\theta, \mathbf{x})$ by $d\mathbf{P}_\theta(X_A)$ which is given by

$$d\mathbf{P}_\theta(X_A) = \exp\left(\int_A \log \lambda(\theta, \mathbf{x}) X_A(d\mathbf{x}) - \int_A \lambda(\theta, \mathbf{x}) d\mathbf{x}\right). \tag{15}$$

2.4. Sequence of Statistical Experiments and Locally Asymptotic Normality (LAN)

The theory of locally asymptotic normality (LAN) was first introduced by Le Cam [2]. The LAN theory provides conceptual simplifications of asymptotic statistics via the existence of a randomized statistic in a limit experiment using convergence in distribution of loglikelihood ratios. It makes possible to construct asymptotically locally most powerful criteria for composite hypothesis test and to establish a minimax bound for the risk of arbitrary estimators [2][3] .

We define the statistical experiment as a statistical model $\{\Omega, \mathcal{X}, \mathcal{P}\}$ where $\mathcal{P} = \{\mathbf{P}_\theta, \theta \in \Theta\}$. That is, the probability distribution of an event $X \in \mathcal{X}$ is one out of the family of distributions $\mathcal{P} = \{\mathbf{P}_\theta, \theta \in \Theta\}$.

Definition 1 *The sequence of statistical experiments $\{\mathbf{P}_\theta^{(n)}, \theta \in \Theta\}$ is locally asymptotically normal (LAN) at θ_0 if there exist matrices $\mathbf{r}_n(\theta_0)$ and $\mathbf{I}_{\theta_0}$ and random vectors Δ_{n,θ_0} such that $\mathcal{L}(\Delta_{n,\theta_0}|\theta_0) \Rightarrow N(0, \mathbf{I}_{\theta_0})$ and for every $\mathbf{h}$*

$$\log \frac{d\mathbf{P}_{\theta_0+\mathbf{r}_n(\theta_0)\mathbf{h}}^{(n)}}{d\mathbf{P}_{\theta_0}^{(n)}}(X^{(n)}) = \mathbf{h}^T\Delta_{n,\theta_0}(X^{(n)}) - \frac{1}{2}\mathbf{h}^T\mathbf{I}_{\theta_0}\mathbf{h} + o_{P_{\theta_0}^{(n)}}(1), \tag{16}$$

where $\mathcal{L}(\Delta_{n,\theta_0}|\theta_0) \Rightarrow N(0, \mathbf{I}_{\theta_0})$ denotes that Δ_{n,θ_0} converges in distribution to $N(0, \mathbf{I}_{\theta_0})$ under $P_{\theta_0}^{(n)}$ and $o_{P_{\theta_0}^{(n)}}(1)$ represents a term that converges to zero in $P_{\theta_0}^{(n)}$ probability.

Here, Δ_{n,θ_0} is called central sequence and $\mathbf{I}_{\theta_0}$ is called Fisher information matrix which actually coincides with the conventional definition of Fisher information for smooth parametric families. The iid drawings of random variables $X_1, \ldots, X_n$ where $X_1 \sim \mathbf{P}_\theta \triangleq \mathcal{N}(\theta, 1)$ gives a good example of LAN family where $\mathbf{P}_\theta^{(n)} = \mathbf{P}_\theta^n$, $r_n(\theta) = \frac{1}{\sqrt{n}}$, and $\Delta_{n,\theta} = \frac{1}{\sqrt{n}}\sum_{i=1}^n (X_i - \theta)$.

When the sequence of experiments satisfies the LAN conditions, we can construct the asymptotic local upper bound of the power and a sequence of tests that achieves the bound [2][3].

Theorem 1 *Let $\{\phi_n\}$ be any sequence of asymptotic α-tests for hypothesis (3). That is,*

$$\limsup_{n\to\infty} \mathbb{E}_{n,0}\phi_n \leq \alpha.$$

Suppose that $\{\mathbf{P}_\theta^{(n)}, \theta \in \Theta = [0,\infty)\}$ is LAN at $\theta = 0$ with normalizing sequence $r_n \to 0$, central sequence $\Delta_{n,0}$, and FIM I_0. Then, for any $M > 0$,

$$\limsup_{n\to\infty} \sup_{0<r_n^{-1}\theta\leq M} \left[\mathbb{E}_{n,\theta}\phi_n - Q(z_\alpha - r_n^{-1}\theta I_0^{1/2})\right] \leq 0.$$

where $\mathbb{E}_{n,\theta}$ denotes the expectation under $P_\theta^{(n)}$ probability. Furthermore, the following procedure is asymptotic α-test for the hypotheses (3) that achieves the bound

$$\text{Take } H_0 \quad \text{if} \quad I_0^{-1/2}\Delta_{n,0} \leq z_\alpha \tag{17}$$

$$\text{Take } H_1 \quad \text{if} \quad I_0^{-1/2}\Delta_{n,0} > z_\alpha, \tag{18}$$

where $\Delta_{n,0}$ is the central sequence and $z_\alpha = Q^{-1}(\alpha)$.

A test that achieves the asymptotic local upper bound with the asymptotic size constraint α is called asymptotically locally most powerful (ALMP) α-test. The meaning of ALMP can be described as as follows. Suppose that the alternative hypothesis is bounded away from zero, $\theta \geq \theta_1 > 0$. As the sample size of observations goes to infinity, two sequences of distributions $\{\mathbf{P}_0^{(n)}\}$ and $\{\mathbf{P}_{\theta_1}^{(n)}\}$ becomes asymptotically entirely separated and any reasonable detector approaches the detection probability of one with possibly different convergence rate. When two tests give the detection probability very close to one for a large sample size, the convergence rate is no longer a proper measure for the performance of detector. Hence, the focus of the detection problem is moved to the parameter range, the amplitude of signal or signal-to-noise ratio range in our case, where the distribution of null and alternative hypotheses are still nonseparable. For the one-sided detection problem, the ALMP detector is most powerful not only in the local neighborhood of null parameter but also in the entire parameter space $\Theta = [0,\infty)$ [4].

3. DETECTION OF SPATIALLY-VARYING SIGNAL

To model a large-scale sensor network, we consider the asymptotic case where the number of sensors deployed over a fixed space A goes to infinity. Under the Poisson regime, this model is described by increasing the initial intensity λ_h of sensor deployment.

Model 1 (Fixed area and infinite sensor model) *The intensity of Poisson process of initial sensor distribution over space A with finite area is given by*

$$\lambda_h = n\lambda_{h0}. \tag{19}$$

Then, for each $n \geq 1$, the local intensity of Poisson distribution of the alarmed sensors is given using (10, 11) by

$$\lambda^{(n)}(\theta, \mathbf{x}) = \theta n f(\mathbf{x}) + n\lambda_0, \tag{20}$$

and the sequence of experiments $\{\mathbf{P}_\theta^{(n)}, \theta \in [0,\infty)\}$ is given by (15). $X_A^{(n)}$ is the realization of Poisson processes of alarmed sensors on area A with probability $\mathbf{P}_\theta^{(n)}$.

Theorem 2 *For Model 1, suppose that $f(\mathbf{x})$ satisfies the following conditions*

(C.1) $f(\mathbf{x}) \geq 0, \ \ x \in A$,

(C.2) $\sup_{\mathbf{x}\in A} f(\mathbf{x}) < \infty$,

(C.3) $\int_A f(\mathbf{x})d\mathbf{x} > 0$.

Then, the statistical model $\{\mathbf{P}_\theta^{(n)}, \theta \in \Theta\}$ is LAN at $\theta = 0$:

$$\log \frac{d\mathbf{P}_{r_n(0)h}^{(n)}}{d\mathbf{P}_0^{(n)}}(X_A^{(n)}) = h\Delta_{n,0} - \frac{1}{2}h^2 + o_{P_0^{(n)}}(1), \tag{21}$$

where the central sequence and normalizing sequence are given by

$$\Delta_{n,0} = \int_{A_n} r_n(0)^{-1/2}\left(\frac{\dot{\lambda}^{(n)}(0,\mathbf{x})}{\lambda^{(n)}(0,\mathbf{x})}\right)\left[X_A^{(n)}(d\mathbf{x}) - \Lambda_0^{(n)}(d\mathbf{x})\right], \tag{22}$$

$$r_n(0) = J_n(0)^{-1/2}, \ \ J_n(0) = \int_A \left(\frac{\dot{\lambda}^{(n)}(0,\mathbf{x})}{\lambda^{(n)}(0,\mathbf{x})}\right)^2 \Lambda_0^{(n)}(d\mathbf{x}),$$

$$\dot{\lambda}^{(n)}(\theta,\mathbf{x}) = \frac{\partial}{\partial v}\lambda^{(n)}(v,\mathbf{x})|_{v=\theta}, \ \ \Lambda_0^{(n)}(d\mathbf{x}) = \lambda^{(n)}(0,\mathbf{x})d\mathbf{x}.$$

Proof) In [14].

Theorem 3 *For model 1, let the conditions (C.1)-(C.3) be satisfied. Then, the spatial function $s(\mathbf{x})$ provides an optimal weighting function in the sense that it achieves the asymptotic local upper bound of the power under given size constraint as the number of sensors goes to infinity.*

Proof)

$$\frac{\dot{\lambda}^{(n)}(0,\mathbf{x})}{\lambda^{(n)}(0,\mathbf{x})} = \frac{nf(\mathbf{x})}{\theta n f(\mathbf{x}) + n\lambda_0}|_{\theta=0} = \lambda_0^{-1} f(\mathbf{x}),$$

$$r_n^{-1/2}(0) = n^{-1/2}\lambda_0^{1/2}\left(\int_A f^2(\mathbf{x})d\mathbf{x}\right)^{-1/2}.$$

$$\begin{aligned} \Delta_{n,0} &= r_n^{-1/2}(0)\int_A f(\mathbf{x})[X^{(n)}(d\mathbf{x}) - \Lambda_0^{(n)}(d\mathbf{x})], \quad (23) \\ &= n^{-1/2}\lambda_0^{-1/2}\left(\int_A s^2(\mathbf{x})d\mathbf{x}\right)^{-1/2} \\ &\quad (\sum_{i:\ \mathbf{x}_i \in A} s(\mathbf{x}_i) - n\lambda_0 \int_A s(\mathbf{x})d\mathbf{x}). \quad (24) \end{aligned}$$

■

Notice that the asymptotic optimal test statistic is the weighted sum of alarmed sensors. The weight $s(\mathbf{x})$ is related to the shape of underlying spatial function $\gamma(\mathbf{x})$. For the proposed method, the optimal test can be implemented in a simple manner without obtaining the exact value of $g(\gamma(\mathbf{x}))$ since the optimal weight is the local intensity variation $s(\mathbf{x})$ of alarmed sensors.

Corollary 1 *For iid sensor observations over A, the counting rule is asymptotically locally most powerful α-test under the Poisson assumption on sensor distribution.*

Proof) In this case, the spatial signal shape $s(\mathbf{x})$ is given by

$$s(\mathbf{x}) \equiv 1,$$

and the central sequence is given by

$$\Delta_{n,0} = (n\lambda_0|A|)^{-1/2}(N^{(n)}(A) - n\lambda_0|A|), \quad (25)$$

where $N^{(n)}(A)$ is the number of alarmed sensors in space A and $|A|$ is the area of A. ■

3.1. Estimation of Optimal Weight Function

As shown in Theorem 3, the optimal weight for the local neighborhood of $\theta = 0$ under Poisson assumption is the spatial signal shape $s(\mathbf{x})$. The spatial variation $s(\mathbf{x})$ can be obtained without measuring the signal strength. (10) reveals that the $s(\mathbf{x})$ is the local intensity variation of alarmed sensor distribution over space. Hence, the weight can be obtained from the alarmed sensors and their location directly. For example, we can model

$$s(\mathbf{x}) = \sum_j s_j I_{A_j}, \quad \bigcup_j A_j = A. \quad (26)$$

Under Poisson assumption, the maximum likelihood estimator for $s(\mathbf{x})$ is given by

$$\hat{s}_{ml}(\mathbf{x}) = \sum_j \frac{N(A_j)}{|A_j|} I_{A_j}. \quad (27)$$

In this case, several measurements by sensors are required for the purposes of estimating weight and providing test statistics.

4. CONCLUSION

Assuming Poisson distribution on sensor locations and the availability of location information, we proposed an efficient way of utilizing the spatial variation of the underlying phenomenon to optimize the global decision under Neyman-Pearson context. We obtained the asymptotically locally optimal test for a large sensor network with nonidentical sensor observation under the Poisson assumption. We proposed a simple estimation of the optimal weight to implement the proposed detector without measuring the exact value of underlying phenomenon.

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