Capacity and Medium Access Control for Large Scale Sensor Networks

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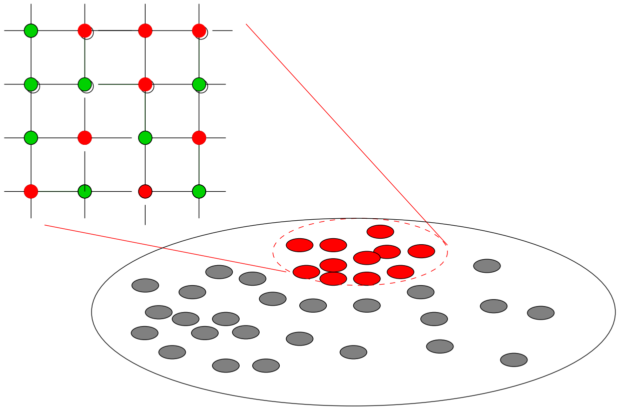
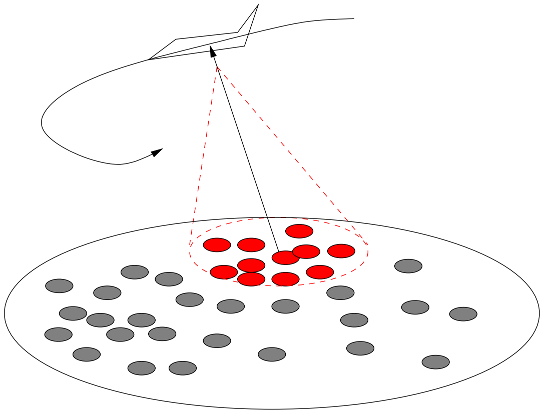
Outline

Capacity and Stability of Regular Networks



- Random reception;
- Fading links with/without channel state information.
- Random duty cycles.

MAC for Sensor Reach Back



- VLSN: very large sensor networks
- Primitive nodes
- Low rate packets
- Mobile access point

Network Capacity

The Capacity Definition

- Nodes always have packet.
- Define $W_{ij}(t)$ as the number of packets from i received by node j
- Rate λ is called uniformly achievable if there exists a scheduling policy delivering packets with rate λ from every node to every other node *i.e.*,

$$\liminf_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} W_{ij}(t) \geq \lambda, \ \forall i,j \in \mathcal{N},$$

  is satisfied with probability one.
- The capacity η is the sup. of such rates.

Related Work

- Gupta-Kumar: general static network $\eta \sim O(\frac{1}{\sqrt{N}})$
- Grossglauser-Tse: network with randomly walking nodes $\eta \sim O(1)$

Network Stability

The Stability Definition

- Packets from i to j arrive randomly with rate λ.
- Define $N_i(t)$ as the number of packets at node i.
- A network is stable if the buffer of every node is stable at rate λ *i.e.,*

$$\lim_{\theta\to\infty} \liminf_{t\to\infty} Pr\{N_i(t) < \theta\} = 1, \quad \forall i \in \mathcal{N}$$

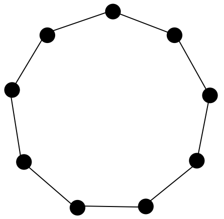
Related Work

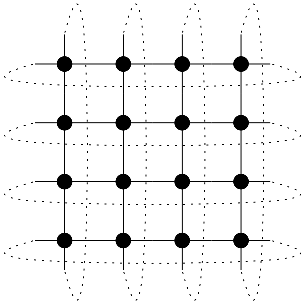
- Sylvester-Kleinrock'83,Tsybakov-Bakirov'85: Stability of ALOHA.
- Tassiulas-Ephremides'92: General topology with centralized control.

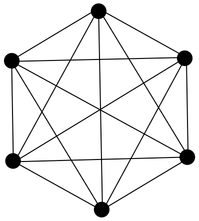
We obtained a general condition when the maximum stable throughput is equal to the network capacity.

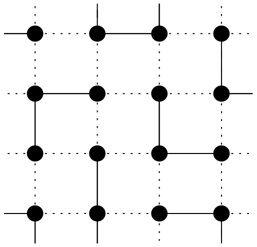
Regular Networks and Reception Models

Regular Networks


a) Ring


b) Manhattan


c) Fully connected


d) Manhattan
with fading links

Reception Model

- Slotted, half duplex transmission/receptions.
- The reception probabilities (Ghez, Verdu and Schwartz '88)

$$C_{n,k} = Pr[k \text{ rec'd} \mid n \text{ tx'd}] \quad \mathbf{C} = \begin{pmatrix} C_{1,0} & C_{1,1} & & \\ C_{2,0} & C_{2,1} & C_{2,2} & \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

Capacity vs. ALOHA Capacity

Theorem: Let $C_n = \sum_{k=1}^{n} kC_{n,k}$ be the average number of received packets given n simultaneous transmissions. Then the capacity of the Manhattan network is

$$\eta = \frac{1}{\sqrt{N}} \max_{i=1,\cdots,4} \frac{2C_i}{i+1} + O(\frac{1}{N})$$

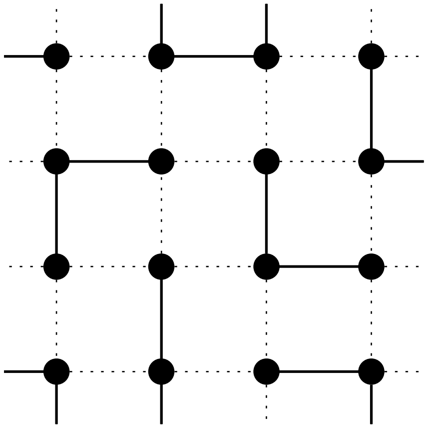
Capacity with the slotted ALOHA is

$$\eta_{ALOHA} = \frac{1}{\sqrt{N}} \max_{0\leq q\leq 1} \sum_{k=1}^{4} \binom{4}{i} q^i (1-q)^{5-i} \frac{C_i}{2}.$$

Comparison: For the K-collision channel

	K=1	K=2	K=3	K=4
$\eta \cdot \sqrt{N}$	1.00	1.33	1.50	1.60
$\eta_{ALOHA} \cdot \sqrt{N}$	0.16	0.34	0.46	0.50

Manhattan Networks with Fading Links



Fading Model

- link fade rate p.
- $\mathbf{C} = \begin{pmatrix} 0 & 1 & & & \\ 1 & 0 & 0 & & \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$.

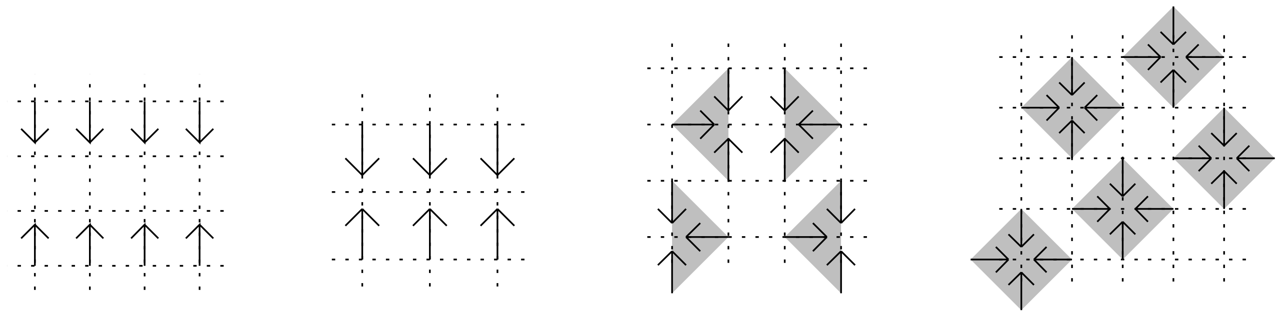
Network Capacity: Let η_* be the capacity with CSI and η without. Then

$$\eta = \frac{1}{\sqrt{N}} \max_{i=1,\cdots,4} \frac{2ip(1-p)^{i-1}}{(i+1)} + O(\frac{1}{N}),$$

Furthermore, as $N \to \infty$,

$$\lim_{p\to 0} \frac{\eta_*}{\eta} = 2.5 \quad \text{and} \quad \lim_{p\to 1} \frac{\eta_*}{\eta} = 1.$$

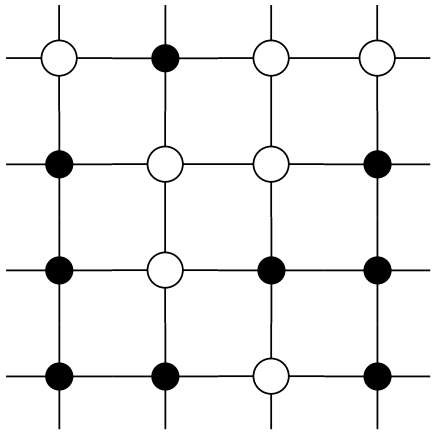
Optimal Medium Access with Fading Links



a) 1−MPR b) 2−MPR c) 3−MPR d) 4−MPR

$$K = \arg \max_{i=1,\cdots,4} \frac{i(1-p)^{i-1}}{(i+1)} = \begin{cases} 1 & 1 \geq p \geq \frac{1}{4} \\ 2 & \frac{1}{4} \geq p \geq \frac{1}{9} \\ 3 & \frac{1}{9} \geq p \geq \frac{1}{16} \\ 4 & \frac{1}{16} \geq p \geq 0 \end{cases}$$

Nodes with Random Duty Cycle



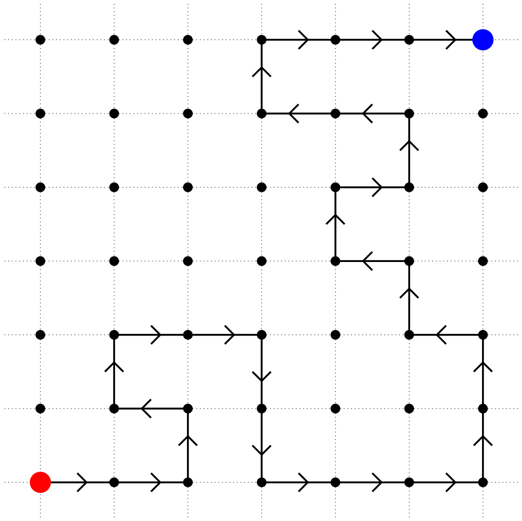
Network Model

- Duty cycle p.
- No node state information.
- $\mathbf{C} = \begin{pmatrix} 0 & 1 & & & \\ 1 & 0 & 0 & & \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$

Network Capacity

$$\eta = \frac{1}{\sqrt{N}} \max_{i=1,\cdots,4} \frac{2ip^2(1-p)^{i-1}}{(i+1)} + O(\frac{1}{N})$$

Network Capacity with Gossiping



The network capacity is

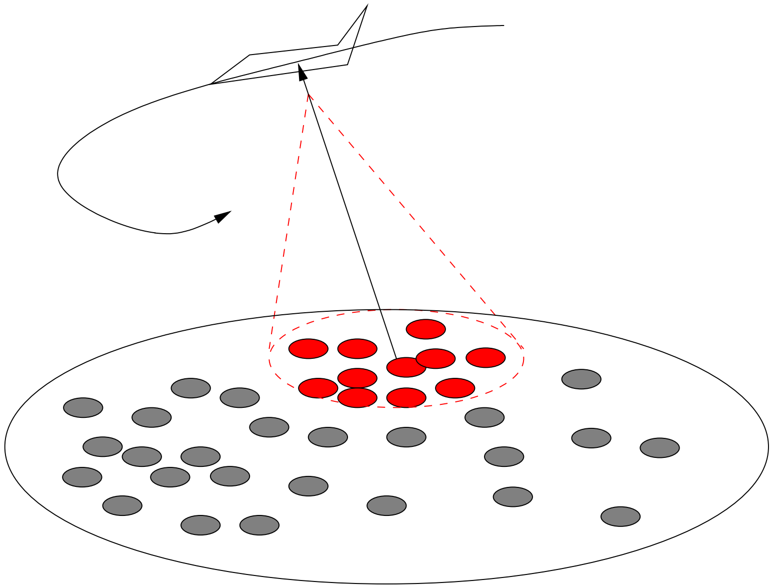
$$\eta = O(\frac{1}{N \log N}).$$

With ALOHA, we have

$$\eta_{ALOHA} = O(\frac{1}{N \log N}).$$

Significantly lower than $O(\frac{1}{\sqrt{N}})$!

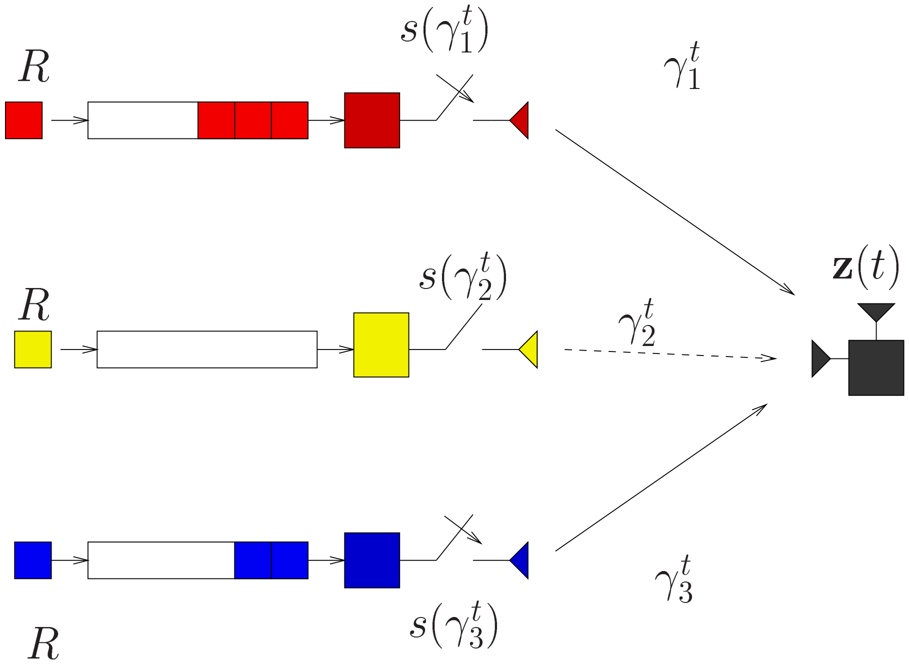
MAC for Sensor Reachback



Sensor Network Characteristics

- Number of nodes is very large.
- Nodes are densely deployed.
- Sensor nodes can failure quite easily.
- Nodes are limited in power, computational capabilities and memory.
- Rapid fading channels
- The nodes may not have global identifiers.

Slotted ALOHA with CSI



Channel Model

- Channel state is i.i.d between users and slots with distribution $F(\gamma)$.

Protocol Discipline

- User i has access to γ_i^t alone.
- User i transmits with probability $s(\gamma_i^t)$.
- The function $s(\cdot)$ is the “scheduler”.

Maximum Stable Throughput

Maximum Stable Throughput

Theorem 1 *The system is stable if* $R < \lambda_M(s(\cdot))$ *and unstable if* $R > \lambda_M(s(\cdot))$*, where*

$$\lambda_M(s(\cdot)) = \sum_{k=1}^{M} \binom{M}{k} (1-p_s)^{M-k} p_s^k C_k(G_s(\cdot)).$$

Transmission Control $s(\gamma)$ affects

- Transmission probability: $p_s \stackrel{\Delta}{=} \Pr[\mathsf{Tx}] = \int_0^\infty s(\gamma) dF(\gamma)$
- The a posteriori CSI distribution:

$$G_s(\gamma) \stackrel{\Delta}{=} \Pr[\Gamma < \gamma | \mathsf{Tx}] = \frac{1}{p_s} \int_0^\gamma s(\gamma) dF(\gamma)$$

- Average # receptions with k transmissions: $C_k(G_s(\cdot))$

An Example of Optimal Transmission Control

The Model:

The transmission of user i is successful iff only user i transmits and its SNR γ exceeds γ_0.

Optimal Transmission Probability

- Denote $p_{\gamma_0} = P\{\gamma \geq \gamma_0\}$ and assume $p_{\gamma_0} \geq \frac{1}{M}$.
- The maximum stable throughput after optimizing scheduler is

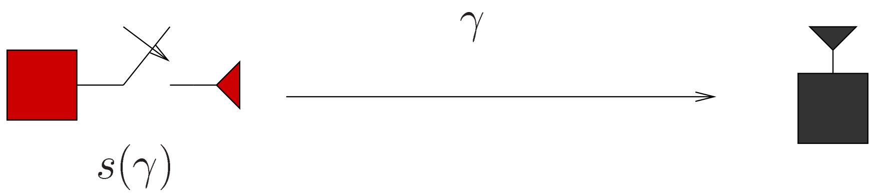
$$\lambda = (1 - \frac{1}{M})^{M-1}$$

- An optimal scheduler is

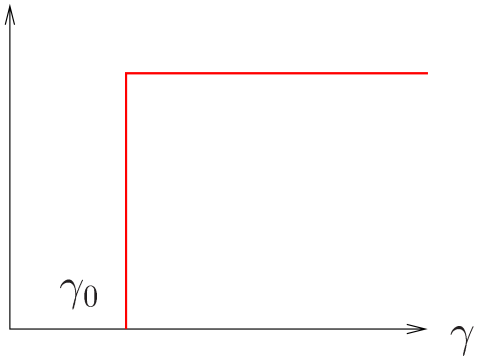
$$s^*(\gamma) = \begin{cases} 0 & \gamma < \gamma_0 \\ \frac{1}{Mp_{\gamma_0}} & \gamma \geq \gamma_0 \end{cases}$$

Shaping the Channel State Distribution

From A Priori CSI to A Posteriori CSI

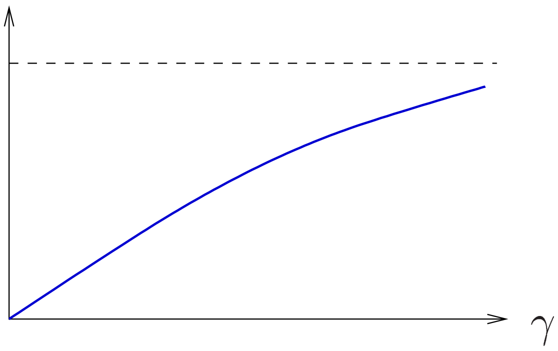


$s(\gamma)\overset{\Delta}{=}\Pr[\mathsf{Tx}|\Gamma=\gamma]$



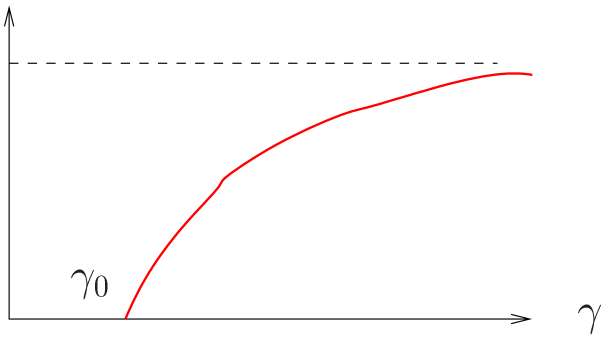
Transmission Control

$F(\gamma)\overset{\Delta}{=}\Pr[\Gamma\leq\gamma]$



A Priori CSI

$G_s(\gamma)\overset{\Delta}{=}\Pr[\Gamma<\gamma|\mathsf{Tx}]$



A Posteriori CSI

Asymptotic Stable Throughput

Asymptotic Stable Throughput

Given $\{F(\gamma), s_n(\gamma)\} \rightarrow \{p_{s_n}, G_{s_n}(\gamma)\}$, define the asymptotic stable throughput by

$$\lambda(\{s_n(\cdot)\}) \;=\; \liminf_{n\to\infty} \lambda_n(s_n(\cdot)) = \liminf_{n\to\infty} \sum_{k=1}^{n} \binom{n}{k} (1-p_{s_n})^{n-k} p_{s_n}^k C_k(G_{s_n}(\cdot)).$$

Relevance

- Metric of interest for “large” networks.
- Easier to find good transmission control algorithms.

Transmission Control With and Without CSI

- The maximum AST with controls that do not use CSI is

$$\lambda^{\infty} = \sup_{x} e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{k!} C_k(F(\cdot))$$

S.Ghez, S.Verdu and S.Schwartz, ‘Optimal Decentralized Control in the Random Access Multipacket Channel”, IEEE Trans. Automatic Control, 34(11):1153-1163, November 1988.

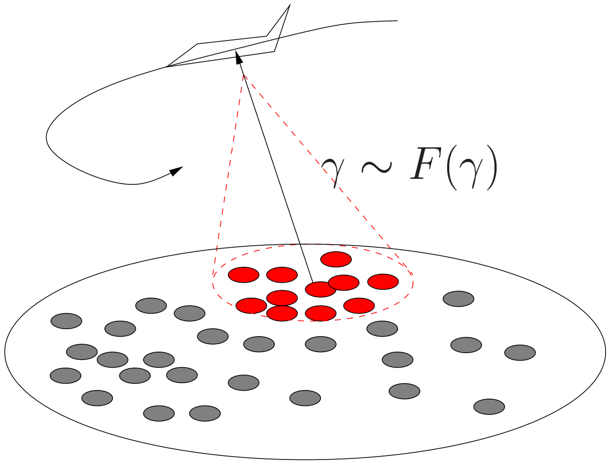
- The maximum asymptotic stable throughput with CSI is

$$\lambda^{\infty}_{\mathsf{CSI}} = \sup_{x, G(\cdot)} e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{k!} C_k(G(\cdot)),$$

where $G(\cdot)$ is any distribution that is dominated by $F(\cdot)$.

- Equivalent to changing the channel state distribution to $G(\gamma)$!

The Reach Back Problem

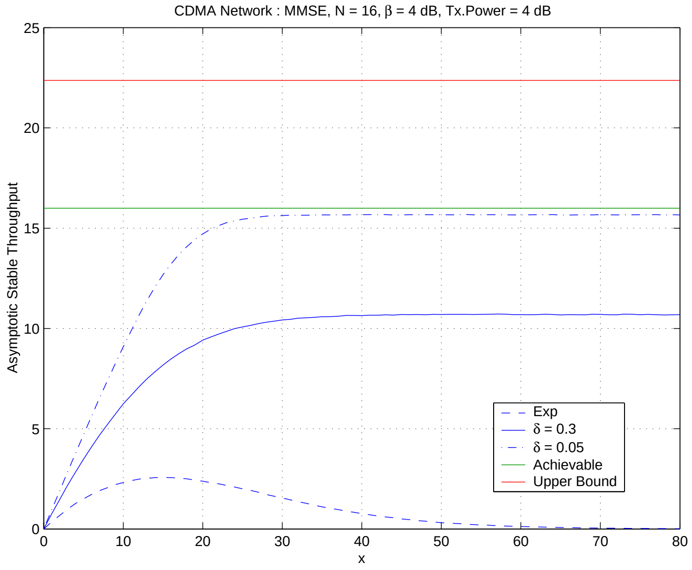


Shaping the Fading Statistics

Good target distributions and trans. probability are

$$1 - G(\gamma) \sim \frac{1}{\gamma^{\delta}}$$

$$s_n(\gamma) = \min\{\frac{e^{\gamma/P_T}}{\gamma^{\delta+1}}\frac{x}{n}, 1\}1_{\gamma>\gamma_0}, \quad \gamma_0 > 0$$



Theorem:

The asymptotic throughput without CSI diminishes as $P_T \to 0$. The asymptotic throughput with CSI is lower bounded by the spreading gain N.

$$N \leq \lambda^{\infty}_{\text{CSI}} \leq N + N/\beta, \quad \forall P_T, \beta > 1$$

$\lambda^{\infty} \to 0$ as $P_T \to 0$

Conclusions

- The asymptotic behavior of network capacity (both the order and the scaling factor) provides insights into MAC and routing designs.
- The PHY property C_{ij} affects the network capacity in different ways. The capacity expression showsthe tradeoff between complexity and capacity gain.
- MAC for VLSN requires different approaches.
- CSI makes a fundamental difference, and cross-layer design matters.